# THE MATHEMATICS TEACHER

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# THE MATHEMATICS TEACHER

Volume XL Number 2

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## Coralling the Wandering Decimal Point[1]


*By* MARY A. POTTER

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PERHAPS you in Northeast Iowa have pupil trouble, such as we have in southeast Wisconsin. One of these difficulties was aired at a pre-school teachers meeting in August by means of the following song sung to the tune of "Good Morning, Mr. Zip."

Good Morning, little Jacks and Jills
With your sloppy Joes and bobby socks.
Good morning, little Jacks and Jills,
With your "Butched-up" curly locks.
Quiet down! Now don't get teacher fussed.
If Sinatra doesn't get you, then Bing Crosby must.
Good morning, little Jacks and Jills,
How about that math assignment
Your physical alignment
Or are we asking far too much?

Deeply engrossed as they are in the radio, movies, much athletics, and the current boy-friend or girl-friend (all excellent interests if taken in moderation), our Jacks and Jills do frequently feel that we are asking far too much. But when they leave school and exchange its sheltered understanding and vivid excitement for the rigors of living in a modern world and the drab realities of a job where they must produce accurate results, the story is different. Too often they arrive at the predicament told in the following UP dispatch:

*Somewhere in Ohio.* (We kindly suppress the actual name.) A taxpayer, who went without things he needed for 3 weeks and then borrowed $10 to pay $62 in taxes, got it all back except 62¢. He had misread the decimal point.

A teacher of mathematics wonders how so trite a paragraph could be broadcast by the UP. It is no man-bites-dog story but a dog-bites-man story of the most common variety. As usual the decimal point had wandered, and, again as usual, it was making trouble. Shall we devote the next few minutes to a discussion of the wandering of the decimal point, why it wanders, where it wanders and what measures we can take to corral it, paying special attention to its wandering when it sees the sign of division.

So constant and disastrous has been this roaming that it has not only attracted the attention of the elementary and junior high school teachers who observe its strange locations and has made the careful senior high school teachers pause and reteach, but it has also been made the subject of research for investigators whose studies corroborate the facts that any thoughtful teacher knows.

A statistician has been defined (by a cynic) as a man who draws a mathematically precise line from an unwarranted assumption to a foregone conclusion. But we recognize that the contribution of investigators with their statistics are important

[1] Read at the Northeastern Iowa Teachers Association, Dubuque, Iowa on September 27, 1946.

in the solution of our problems, so we shall briefly review their recent contributions.

Brueckner[2] pioneered in the study of decimals in a work he reported in 1930. He listed errors in the four fundamental processes in decimals under three main headings: Difficulties basic to the process, Difficulties peculiar to decimal situations, and Other Difficulties. He summarized his findings with the observations that

1. Many pupils do not have adequate concepts of the numerical value of decimals and

2. In addition and subtraction the major difficulty was the misplacing of the decimal point; in multiplication and division misplacing of the decimal point shared honors with the error of omitting it completely.

Evidently in the intervening years these errors have not been generally corrected, according to an article on "Weaknesses in Arithmetic Teaching" by Henderson reported in the June, 1946 issue of the *Elementary School Journal*. In discussing the training program for pilots of the Army Air Forces he says: "Those of us who were teaching the mathematics course often commented on the shaky foundation in arithmetic possessed by many of the cadets. . . .

> The first glaring weakness was inaccuracy in the four fundamental operations of arithmetic. Of these, division was worst, particularly division involving decimals. It was astounding how many cadets were ignorant of any reliable method for establishing the decimal point in the quotient. Many who said they placed the point by rough estimate, could not estimate the magnitude of the quotient with any degree of confidence.

The observation of most teachers agrees with that of Henderson that division of decimals is the most difficult process where the greatest number of errors occur. Grossnickle, who has contributed so ably to teaching division of whole numbers by a one-figure divisor, has made investigations of division of decimals which you may wish to read for yourself. One article called "Types of Errors in Division of Decimals" appears in the Elementary School Journal for Nov., 1941; "Kinds of Errors in Division of Decimals and Their Constancy" is found in the Oct. 1943 issue of the Journal of Educational Research; and "Some Factors Affecting a Test Score in Division of Decimals" in the January 1944 issue of the same magazine. His listing of 21 types of errors is especially good and includes 5 due to placement of the point and 5 caused by shifting the point.

If we can learn why the decimal point wanders, we may be more able to contrive methods of keeping it in its proper place.

Possibly the first indifference about the decimal point begins in the primary grades. When the child first meets the decimal point he is introduced to it as the *cents' point*, which is a necessary and at times helpful method of approach. But, so very often the bridges connecting dimes and tenths or those from cents to hundredths are not pointed out clearly or successfully crossed, and much of the value that our decimal monetary system could bring to the teaching of decimals is lost.

In addition, small amounts of money may be written by using the symbol for cents with no point, and at other times using the symbol for dollars with the decimal point—both forms necessary for the child and adult to read and use. But quite frequently Jack and Jill do not understand why the cents' point is included or omitted, so when they write a money sum, the presence of the decimal point with either the symbol for dollars or cents does not show the amount of money being considered, but rather the degree of care that the writer is using. This uncertainty remains when Jack becomes Mr. Smith. Is it one of your pet peeves to have to pay 13¢ for 3 bunches of radishes which Mr. Smith the grocer has carefully marked as "3 bunches for .13¢"?

Another difficulty with the decimal point is that its importance has not been duly stressed. Even in this democratic age

[2] Brueckner, Leo. *Diagnostic and Remedial Teaching in Arithmetic*, Winston, Chicago, 1930, pp. 228–237.

we must confess that the decimal point is a despot; as king of the number its very position determines the value of the number in which it resides. The other punctuation mark found in numbers, the comma, is a weak brother used as a crutch for easy reading, whose presence or absence does not affect the value of the number. Many children consider that the period and comma in a number have equal importance. As we understand their thinking, we see why the decimal point, imitating the comma, may be found two or three times in a number, or that it may be omitted altogether as is the comma, if that is more convenient.

Possibly another reason that the decimal point is prone to wander is that decimal notation—the place value of a number—is as difficult for Jack and Jill to understand completely as it is easy for them to use when this understanding is accomplished. We know that Euclid finished his writing of the famous Elements many centuries before Arabic numbers were commonly used in Europe. Since it took the scholars of the human race until 1600 A.D. to invent the writing of fractions in decimal form, perhaps it is not surprising that Jack and Jill, who will never be scholars and who study arithmetic a very few hours a week, find that decimal fractions are a stumbling block. Moreover, I have yet to hear of a teacher in America who has been motivated by the great enthusiasm for decimals displayed by one Robert Norton who translated into English the first exposition of decimals. He was so impressed with their value that he offered to go around England teaching decimal fractions free of charge as a patriotic service.

Yet another reason that the decimal point wanders is that much teaching, especially of division of decimals, is done by rote. Jack and Jill are instructed to move the decimal point two places to the right or three places to the left, according to some incantation that the teacher chants and which the pupils are inclined to disregard. After the decimal point is safely moved, other operations take place and the right answer is produced. Jack glows with the satisfaction of a job well done, but he rarely understands thoroughly the mysteries that have produced that successful answer. Two years later when he again needs to divide decimals, he remarks "I don't remember how you do it exactly, but you move the decimal point!"

Perhaps a clue to the fact that the pupils do not understand shifting of decimal points in division may be found in a study of the professional training of elementary-school teachers which was made by Robinson.[3] He found that 70% of them did not know what fundamental principle is illustrated in the step in a division example: $.6\overline{)\$25.44} = 6\overline{)\$254.40}$.

Since at least part of the wandering of the decimal point can be laid to the shifting commonly used in division, it is fitting that we examine carefully various methods used in division to find out if we can teach the process with meaning to produce correct results without encouraging the making of new errors.

You may be interested to know that the original treatise of 1600 A.D. in its explanation of division of decimals made use of the subtraction method based upon a reverse of the method used in multiplication when the number of decimal places was added.

Thorndike[4] explains division using the subtraction method. Morton[5] recommends that when the divisor is a whole number the decimal point in the quotient should be put directly above the decimal point in the dividend; when the divisor contains a deci-

[3] Robinson, Arthur E. *The Professional Training of Elementary School Teachers in the Field of Arithmetic*, p. 22. Teachers College Contribution to Education, No. 672. New York: Teachers College, Columbia University, 1936.

[4] Thorndike, Edward L., *Psychology of Arithmetic*. New York, The Macmillan Company, 1924, p. 271.

[5] Morton, Robert L., *Teaching Arithmetic in the Elementary Schools*, Vol. II. New York, Silver Burdett Company, 1938, p. 344; pp. 353–355.

mal, he advises the subtraction method.

Wheat[6] and Wilson[7] advocate the shifting of the decimal point.

But for 30 years or more almost all texts and teachers have moved the decimal point, or taught an adaptation of that method by inserting carets. The advantage of this method is that it is, or usually becomes, as easy rote way to teach the process, but all rote teaching is simple.

Recently the question of method to use has been the subject of a controversy in a series of articles in THE MATHEMATICS TEACHER. In the February 1945 issue Brown of Central Missouri State Teachers College concludes:

> In teaching division of decimals as in all arithmetic teaching, emphasis must be on understanding and not on mechanical techniques. Hence, as these facts indicate, the subtraction method of locating the decimal point in the quotient is preferable and should be taught either in addition to, or instead of, the caret method.

In THE MATHEMATICS TEACHER of October, 1945, Van Engen explains his disagreement with Brown.

In the April, 1946 issue of THE MATHEMATICS TEACHER Miss Mary Crofts tells of an experiment made by 73 teachers in the public schools in Buffalo which resulted in the changing of the teaching in their city from the caret method to the subtraction method, the inverse of multiplication.

John T. Johnson, in the May 1945 MATHEMATICS TEACHER, wisely suggests that the question of the method of teaching division of decimals be decided scientifically by carefully set-up studies. Meanwhile children have to be taught and, until undisputable evidence can be produced that one specific method is correct, we shall have to rely upon our experience, and that of others, upon our good judgment and our present knowledge of the psychology of learning.

[6] Wheat, Harry Grove, *The Psychology and Teaching of Arithmetic*. Boston, D. C. Heath and Company, 1937, pp. 412–415.

[7] Wilson, Guy M., Stone, Mildred B., and Dalrymple, Charles O., *Teaching the New Arithmetic*, New York, McGraw-Hill Book Company, 1939, p. 220.

That the shifting of decimal points in division is ineffective I first observed when I taught three classes a day in 9th grade business arithmetic where I could measure the amount of skill retained by the pupils who had been taught by many different teachers with the various methods.

The chief disadvantages of moving the decimal point seem to be:

1. Children lose respect for the placement of the point if it can be moved at will and for no reason that they understand clearly.

2. Only a small per cent of the pupils do understand the reason that it may be moved so clearly that they remember that reason for one year, let alone all the years that they will use decimals.

3. The slide rule is being used increasingly to shorten computation in business, industry and engineering. Any method which depends upon placement of written figures, as this does, is useless for work with the slide rule.

4. Moving the decimal point gives rise to additional mistakes which do not occur unless this method is used. Although Grossnickle advises that the divisor should always be made a whole number by multiplication, he lists these "Errors Resulting from Shifting of Point."

   a. Shifting when divisor was an integer
   b. Shifted correctly in dividend, point wrong in quotient
   c. Shifted places from left of dividend instead of from right
   d. Shifted incorrectly (number of places)
   e. Shifted to left in dividend

5. This method does not make use of similar knowledge gained in multiplication of decimals. Mathematics is an organized, logical body of knowledge. During the entire build-up of the arithmetical processes it has been customary to teach division as the reverse process of multiplication. For instance, we teach that $7 \times 8 = 56$ and at the same time, or shortly afterwards, that $56 \div 8 = 7$,—to the greater understanding and swifter mastery of both facts.

Multiplication of decimals is much more

difficult to teach than the multiplication facts; to neglect to base the teaching of division upon the hard-won skills and understanding acquired in learning multiplication of decimals is missing an excellent opportunity to enlarge and capitalize related skills and understandings recently mastered, which is good teaching technique. Moreover, not to base teaching of division upon multiplication of decimals is a breakdown in the logical development of number.

6. Moving the decimal point in division, as Jack and Jill actually do it, is usually performed as an artificial trick without meaning. It is probably fortunate for the development of the science of teaching of mathematics that it has received so much adverse criticism because we have been forced to stop and take stock of our teaching techniques. Almost all present-day leaders in the teaching of mathematics have accepted the *meaning theory*, thus discarding the opinion formerly held by most educators that, no matter how achieved, the *results* of learning are of paramount importance. They agree that the children should be made to understand *what* they are to do, *how* and *why* they are to do it. We believe that this principle applies to the teaching of division of decimals as well as to the teaching of other topics.

For the sake of argument, suppose we do decide to divide by the method of shifting the decimal point. To make the development of multiplication logical, we would need to move points in multiplication as well, to make the multiplier a whole number. Shall we work an easy multiplication exercise $1.3 \times .12$ by this method?

To make the multiplier 1.3 a whole number, we must multiply 1.3 by 10, shifting the decimal point one place to the right. In order not to change the value of the product, we must likewise divide by 10, which operation must be performed on the multiplicand, .12, shifting the point one place to the left and making it .012. Our former problem

$$\begin{array}{r} .12 \\ \times 1.3 \\ \hline \end{array}$$

becomes

$$\begin{array}{r} .012 \\ \times \quad 13 \\ \hline \end{array}$$

The decimal point in the product can be placed immediately below that in the multiplicand, thus

$$\begin{array}{r} .012 \\ \times \quad 13 \\ \hline \\ \\ \hline \\ \cdot__ \end{array}$$

Then we multiply to fill in the three places and find the correct product, .156.

This method, of course, is absurd. The children would not remember that multiplying and dividing by 10 is the same as multiplying by 1, they would become confused by the shifting of the points; they would gain nothing by making the multiplier a whole number and by placing decimal points under each other as in addition and subtraction, which act of keeping decimal points over each other or under each other is used as a magical open sesame to substitute for understanding.

But absurd as it appears, this build-up of multiplication is the same development that we expect in division when the point is shifted.

If we have learned to fear that moving the decimal point results in its wider wandering, may we outline briefly another method of teaching division of decimals as the inverse of multiplication, based upon meaning and logical reasoning, which has proved to be effective with many Jacks and Jills?

The teacher has a right to assume that, at this point in his mathematical career, Jack has mastered the meaning of the decimal fraction, nicknamed decimals,

and the three other fundamental operations with them. If the pretest he takes proves that this assumption is optimistic, he must be retaught the basic concepts and the three processes motivated by the many common uses of decimals.

This reteaching may include a review of the place value of whole numbers; the fact that decimals are members of the fraction family but their numerators only are written and their denominators are shown by the number of decimal places used (the meaning of the phrase "number of decimal places" may have to be illuminated); a showing that the denominator of a decimal is a power of 10, although you cannot use the convenient word "power"; the names and values of the decimal places; the relation between common and decimal fractions. Jack also must be able to read decimals with understanding, write them correctly, round them off to a designated degree of accuracy.

If the concept of decimal fractions is really understood, such wrong addition as $.6+.3+.5=.14$ will disappear from the papers as a sum absurd in meaning even to the pupils.

Performing various money transactions and working a set of exercises first with common fractions and then translating them into the decimal fractional form, are probably the best methods of introducing addition, subtraction and multiplication. With a little guidance, rules will suggest themselves to the pupils as short cuts for the processes that are then understood.

In the same way, if division is begun by dividing by a whole number a money sum expressed as a decimal, Jack and Jill feel a confidence resulting from long successful practice.

The next easy step is to find an approximate answer in division of mixed decimals. Suppose that 16.1 is to be divided by 8.05; the numbers are rounded off to the nearest whole number and an approximate quotient is found. $16.1\div8.05$ becomes $16\div8$, which of course is 2. The attention of Jill is focused upon the main part of the number so that division is performed with easy whole numbers, unencumbered by a dragging tail of decimals. Jill is absorbing the idea that $16.1\div8.05$ is about 2.

Another common sense step is to present a set of division exercises followed by 3 possible quotients. Jill is to pick from these three numbers the one that is correct. If she understands rounding off numbers, she will have little difficulty in deciding that the correct answer for the problem ($31.27\div5.9=.53$, or 5.3, or 53) is 5.3 because $30\div6$ is about 5, not about $\frac{1}{2}$, or about 50.

Jack and Jill are now ready to be shown again that dividing is the opposite of multiplying and are given a set of completed simple multiplication exercises to convert into exercises in division. They find that $.2\times3=.6$ can be made into $.6\div2=3$ or $.6\div3=.2$. They review the fact that the number of decimal places in the product is equal to the number of decimal places in the multiplier plus the number in the multiplicand. They examine their quotients in the newly made division to find if there is a relation between the number of places in the dividend, divisor, and quotient, and are led to conclude that probably the number of places in the quotient is found by subtracting the number in the divisor from the number in the dividend.

The conclusion tentatively formed by the previous experiment is reinforced by a carefully prepared set of exercises in which easy decimals are divided by changing them to common fractions.

The fact that the number of decimal places in the quotient is equal to the number in the dividend minus the number in the divisor is crystallized in the form of a rule which is then memorized.

In the next easy stage of development, the division of the decimals has been done for the children while their sole task is to place the decimal point in the correct place in the answer in accordance with the newly developed rule.

This is followed by the final step in division when the whole process is performed

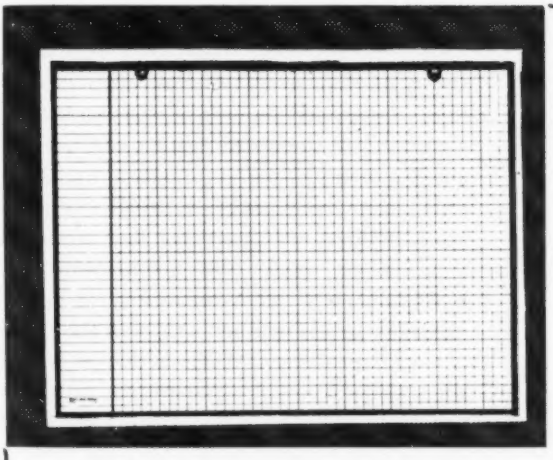
by Jack and Jill, who use the ever helpful multiplication as a check.

Of course attention has to be called to the case of the missing zeros when a whole number is divided by a decimal such as $9 \div .75$ and this case needs adequate practice. A common sense approach shows the absurdities that would result if zeros were not annexed to make as many decimal places in the dividend as there are in the divisor, and works the same problem using common fractions, giving convincing explanations of this troublesome case.

We are now ready for the stage in teaching that is likely to occur in almost any classroom in which the pupils do not understand the work and which in our preschool institute our teachers described in another song sung to the tune of "Sweet and Low":

Repeat again, drill some more,
Johnny can't understand;
Once more, then again—
Susie still waves her hand!
Mary still has that puzzled look
Bobby's got comics behind his big book,
Janie is writing a note—
While another one,
While some naughty one,
Sleeps and sleeps.

In closing may we wish for your pupils that they achieve success in their pursuit of the wandering decimal point; that, as they conquer it, they may learn to *respect* its power over numbers; that they may *understand* its power over numbers and learn the laws under which it operates; that they may erect the fences of understanding which wipe out wandering; that they may become the complete master of the decimal point.

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# Mathematics in General Education

## Are We Building the Ability to Use Quantitative Concepts Actually Needed in Adult Life

*By* HARRIETTE BURR, *Past President, California Mathematics Council*
*San Jose High School, San Jose 14, California*

*Our present mathematics curriculum meets the needs of only 5% of our population: the scientist and engineer.*[1]

Yet every member of society needs a form of mathematics: the quantitative interpretations of consumer and social problems. It is not enough to consider only the vocational needs for mathematics. In curricular planning there must also be considered the quantitative interpretations needed by the adult as a consumer, as a citizen in a democracy, and as a cultural being.

The few quantitative interpretations now presented are usually given in the 8th and 9th grades: 5 to 10 yr. before the student actually experiences these problems in his own life. As a consequence these problems are often highly abstract to him. To postpone all consideration of such problems until the 12th grade would eliminate experience in quantitative thinking for students who terminate their formal education before that grade.

In addition to studying quantitative interpretations of current problems, the student must be sensitized to the existence of quantitative relationships. Too often emotions block full understanding of the social problems of the "other half of society." Only facts can break such emotional sets. Thus, our students need experience in seeking and recognizing all quantitative possibilities of a problem; and they need experience in locating and using data in a quantitative way so as to reach an indisputable conclusion.

[1] Quotation from a paper read to the Annual Conference of the California Mathematics Council, December 27, 1943 by Frank N. Freeman, Dean, School of Education, University of California.

### WHAT QUANTITATIVE INTERPRETATIONS ARE NEEDED BY EVERY ADULT?

The mathematics of general education —the core to be developed for each student—should consist only of those concepts of use to each member of society. Because only a few have any real use for a quadratic equation, this would not be included in the mathematics core. On the other hand, every person must choose between renting or owning his house, buying term or ordinary life insurance, voting for socialized medicine or private practice, etc.

Heretofore, most consumer and social issues have been left to the social studies, home arts, and commercial classes for discussion. It is the responsibility of the social studies to develop an understanding of the part an activity plays in the life of mankind: its historical, cultural, and economic significance. It is the responsibility of mathematics to develop ability to recognize and use quantitative data in the study of social problems. After a discussion by the social studies class, a student often has the same attitude toward the issue as at the outset: a conclusion based on his parents' beliefs and emotions, rather than on the many phases of the issue presented in class. But a quantitative interpretation is indisputable. For this reason all students need development in the use of quantitative interpretations of social issues.

This has been illustrated vividly[2] by a 9th grade class in a community of wealthy business and professional people in the

[2] Kinney, Lucien B., School of Education, Stanford University. Quoted from a discussion at the Annual Conference of the California Mathematics Council, December 29, 1942.

Chicago area. The class had just come from social studies where they had completed a unit on social security. The verdict of the class was, "If these people would only save out of each pay check, they wouldn't need an old-age pension." Their mathematics teacher then had these students budget the income of a family at the standard of living of "minimum health and efficiency." The students were astounded to find that there simply wasn't enough money for any appreciable savings after shelter, food, clothing, and medical expenses had been paid. These same students changed from opposition to wholehearted support of the social-security program.

Following is a list of some of the topics needed by all students, but not found in the usual mathematics textbook. This list cannot be complete. It must be revised continuously as further needs arise.

1. *Maps, latitude, longitude, and time*
   In the Air Age it will be necessary to handle these terms with discrimination. Today's confusion is due to the fact that there has been no explanation of HOW maps are projected, of WHY latitude, longitude, and time are measured as they are. These are topics of solid geometry and astronomy; but they are so simple that a junior high school student can understand them with a careful explanation. Small wonder the average person is confused when the mathematics teacher has ignored these as needed quantitative concepts!
2. *Social security*
3. *Socialized medicine*
4. *Federal support of education*
5. *Free college education for deserving students*
6. *"Boom and Bust" cycles*—what proportion of the people are forced below the standard of living of "minimum health and efficiency" by inflation? Who benefits by inflation? To what extent? What is the effect on foreign countries?
7. *War debt*—how will it be paid? how much is it per U. S. family now? How will fluctuating prices affect the laborer? the clerical worker? etc.
8. *Unemployment*—what are some ways to prevent it?
9. *Wages keyed to cost-of-living indices*
10. *Education*—needs for better education and their costs.
11. *Work week of 30 hours*—health needs of nation indicate greater outdoor recreation—home responsibilities of typical single or married woman worker add approximately 30 hours of *inside* work to the job work week—maintenance of present standard of living if each person is employed only this length of time.
12. *Choice of a vocation*
    Competition in a vocational field: what is the relation between supply and demand? How do the abilities of the individual compare with those of others aspiring to this field?
    Cost of training and possibilities for part-time employment.
    Median remuneration and living standard necessary.

It is not enough to give our future scientist or engineer only the algebra-trigonometry sequence. Neither is it enough to require algebra and geometry of our college-preparatory students. These are the potential leaders of society. Theirs will be the responsibility of making decisions, and of working for support of these decisions. They must have experience in studying the quantitative aspects of consumer and social problems not now included in their curriculum, of understanding the needs of all types of citizens. As leaders they must be able to make just and wise decisions which will be fair to all levels of society. This means a greater allotment of time to consideration of quantitative interpretations.

Thus, an extended curriculum in mathematics is needed so that the many social and consumer issues may have time for consideration. Only the superior student

has a mastery of computation by the end of the 8th grade. To build the mastery necessary for vocational needs as well as for quantitative interpretations will require at least one year beyond the 8th, and for half of the students at least two years. To build an ability to recognize quantitative aspects of social problems and to select crucial data which will lead to a wise interpretation will take at least another year. This means a three, or two-year curriculum between the 9th and 12th grades for consideration of the mathematics needed in general education. Not all students would cover this material at the same rate. The potential scientist and engineer could master this same content in one semester after the 8th grade. The main point is to be sure that each pupil has an opportunity to make quantitative interpretations of these many social and consumer issues, whether it takes one semester or three years.

### Is the Present Grade Placement of Quantitative Interpretations Satisfactory?

At present most pupils receive their last school experience in quantitative interpretations of social and consumer problems in the 8th or 9th grades. Yet, few pupils are sufficiently mature at 13 or 14 years of age to appreciate the full significance of the cost of supporting a family, the effect of socialized medicine, etc. How much more significant would be these problems if they were presented in the 11th and 12th grades!

The three courses which treat consumer and social problems quantitatively are elective and consequently taken only by a small proportion of the students: commercial arithmetic, senior social problems, and home arts courses. Moreover, commercial arithmetic gives its main emphasis to business computation; and home arts courses are restricted to consumer problems. Thus, these courses do not provide the broad coverage needed.

An 8th grade boy has no immediate thought of buying a car. A 10th grade boy with a job is eagerly saving for a car. How much more meaningful would be the unit on the cost of car ownership in the 10th grade than in the 8th grade! Similarly the average girl in the 8th grade has little interest in making a family budget. But 12th grade girls often face the immediate problem of establishing a home. How vital would be a unit in the 12th grade on the cost of maintaining a family!

The college-preparatory student who starts the algebra-trigonometry sequence in the 9th or 10th grade has no further consideration of quantitative interpretations of social or consumer problems. Neither has the industrial arts, home arts, fine arts, or agricultural major. Each of these students must fully appreciate his adult problems when he is only 13 or 14 years old.

At the same time the terminal student must be considered: i.e. the student who does not attend regular school after the 10th grade. If all consideration of these social problems is held until the 11th or 12th grades, he will miss the discussion completely. Yet of all the students considered, this terminal student needs most to experience this discussion. He has the least innate ability to think through these consumer and social problems for himself.

### Are Our Present Methods the Best Possible in Presenting Quantitative Interpretations?

Time after time the mathematics teacher is cited as an example of poor teaching. Unjustifiably? When the unit of taxes is taught, do we assign each page in the textbook as we come to it? Or do we have the students build their own unit and merely use the book as a reference? For example, students could be asked to list improvements which they would like in their school or community. Then each student could estimate the cost of the project and determine the added assessment which would be necessitated for a home owner, a rancher, a businessman, an industrial concern. Not until a topic is a part

of the life of an individual student will that topic be of value to him. Teachers should help each student find some way in which a topic applies to his own life now.

### How Can We Improve this Situation?

A developmental three-year curriculum in the 9th–12th grades should meet many of the needs discussed. Insurance in the 8th grade, then again in the 9th grade? No! Instead, this strand, or topic of subject matter, should be developed from the 8th through the 12th grades: accident insurance in the 8th grade, health insurance in the 9th grade, automobile insurance in the 10th grade, and life and fire insurance in the 12th grade. In this way the phases of direct application at a particular age level would be studied as close as possible to that age level. Similarly each consumer and social issue would be developed from grade to grade, not given in its entirety at one grade.

In such a curriculum, the terminal student would have studied at least some of the most important phases of each topic. The average student would have had time for the multitude of social and consumer problems which he will experience as an adult. All students would study these problems just before they face them as real issues in the working world, not when they are 13 or 14 years old. Special time limits would enable the superior student to cover such a curriculum in one semester instead of the two or three years needed by the average student.

Such a curriculum would be based on the needs of every student, not on the needs of 5%. The mathematics curriculum is direly in need of revision so as to meet actual student needs. If we as mathematics teachers do not take this step, the trend of the 1930's toward decreasing quantitative experiences will continue. Are the mathematics teachers willing to meet the challenge of putting forth the effort and time to analyze these needs and build such a curriculum?


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# So-called Shortcuts Often Cause Confusion

*By* Edith L. Mossman
*Richmond Union High School, Berkeley 4, Calif.*

In arithmetic through the eighth grade and in first year algebra, is not the thorough understanding of fundamental principles of *first* importance? That this need of *first* importance has not been generally taken care of, is evidenced in many ways: (1) Such reports as that given by Admiral Nimitz, pointing out the weakness of our boys in junior and senior high school mathematics. (2) J. Kadushin's statements about the inability of men in the factories to handle simplest work in fractions, and their fear of taking any course in mathematics. (3) Constant complaint from teachers of physics, chemistry and algebra theory as to ignorance of the formula: what it is, what can and cannot be done to it. (4) The experience of much tutoring going on in universities, showing that great numbers have trouble with college mathematics because they did never really understand their work in arithmetic and algebra.

Every once in a while during the past years, attention has been called to the fact that this confusion about simple fundamentals is partly, although not mainly, due to presenting shortcuts *before* the underlying principles are thoroughly understood and *before* there has been sufficient practice in applying the principle as such, so that it is really established in consciousness.

Many teachers feel that the word "cancel" should never be used until after tenth grade. Tragically common indeed, is the mistake of simply crossing out numbers that look alike, regardless of disobeying mathematical laws, and so producing such absurdities as these:

(1) $\frac{\cancel{9}}{11}+\frac{5}{\cancel{9}}$ (2) $\frac{2}{\cancel{3}}\times\frac{1}{\cancel{3}}$ (3) $\frac{5}{\cancel{6}}\div\frac{\cancel{6}}{7}$

(4) $5\frac{\cancel{2}}{\cancel{3}}\times\cancel{2}\frac{1}{\cancel{3}}$ (5) $\frac{\cancel{4}+b}{\cancel{4}+c}$

(6) $\frac{ac+d}{3b+c}\cdot\frac{ab}{a+b}$.

Then because cancelling is usually a meaningless shortcut with no realization that it involves *division,* the quotient *one* is lost sight of:

1. $\frac{\cancel{3}}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{3}}$ is given as 0 instead of 1
2. $\frac{\cancel{a}}{\cancel{b}}\cdot\frac{\cancel{b}\cancel{c}}{\cancel{c}^2}\cdot\frac{\cancel{a}}{\cancel{c}}$ is given as 0, instead of 1
3. $\frac{\cancel{7}}{\cancel{7}}\times\frac{\cancel{7}}{\cancel{7}}\times\frac{\cancel{7}}{7}$ is given as 7, instead of $\frac{1}{7}$.

This confusion is easily cleared up if the student says, "Divide both numerator and denominator each by 6," or "by 5," or "by a," or "by b," etc. Then it is wise to show the quotient "1" in each case.

$$\frac{\begin{matrix}1\end{matrix}}{\begin{matrix}\cancel{3}\end{matrix}}\times\frac{\begin{matrix}1\end{matrix}}{\begin{matrix}\cancel{3}\end{matrix}}=\begin{matrix}1.\end{matrix}$$
$$\begin{matrix}1\end{matrix}\qquad\begin{matrix}1\end{matrix}$$

Would it not be well to impress strongly upon the young people that in performing any mathematical computation one is (1) adding, (2) subtracting, (3) multiplying or (4) dividing, never anything else than one of these four? In what has been called "cancelling" there is really *dividing* numerator and denominator by the same number.

How many understand why dividing numerator and denominator by the same number keeps the value unchanged? $3/6 = \frac{1}{2}$. Only *one third* as many pieces, i.e. *one* instead of *three;* but each piece *three times* as large, i.e. *one half* instead of *one sixth.*

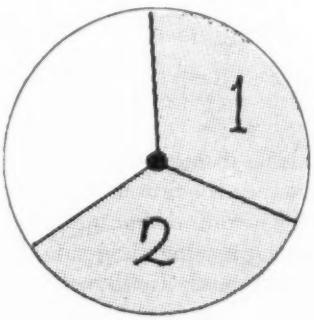
Then the converse of changing fractions to those of higher denomination as when getting ready to add or subtract, almost everywhere in the schools uses a shortcut that carries no meaning in the thought of

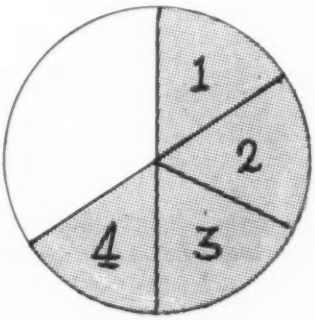
the vast majority. $\frac{2}{3}+\frac{1}{2}=?$ $6\div3=2$ and $2\times2=4$ then $6\div2=3$ and $3\times1=3$. This is followed mechanically.

Instead, after the need for a common denominator, and how to recognize the least common denominator is made plain, the steps should be: What must the denominator 3, be multiplied by to get the common denominator? Then the numerator must be multiplied by the *same* number.

$$\begin{array}{l|ll} 2\times & 2 & 4 \\ & - & = - \\ 2\times & 3 & 6 \end{array}$$

Why is the value of the fraction $\frac{2}{3}$, unchanged? *2 times* as many pieces, i.e. 4 instead of 2, but each piece only *one half* as large, i.e. a sixth instead of a third.




If this plan is followed in addition and subtraction of fractions in algebra, much grief is avoided. For example, in combining

$$\frac{x+2}{x-3}-\frac{x+3}{x-2}+\frac{1}{x}$$

$$\text{L.C.D.}=x(x-3)(x-2).$$

Then we have

$$\frac{(x+2)\cdot x(x-2)}{(x-3)\cdot x(x-2)}-\frac{(x+3)\cdot x(x-3)}{(x-2)\cdot x(x-3)}+\frac{1\cdot(x-3)(x-2)}{x\cdot(x-3)(x-2)}.$$

Look at the denominator of the first fraction. Look at the least common denominator. By what must the first denominator be multiplied to make the common denominator? Then the numerator must be multiplied by the *same* factor, to keep the value of the fraction unchanged, etc.

The next "shortcut" to be discussed is one whose weaknesses and bad effects have been pointed out many times during the last twenty years or so, and still there are those so short-sighted as to use it like candles, when another method like electric lights gives far more illumination to all, at the same time avoiding much confusion.

In some class rooms, when the equation is first introduced, there is discussion of "keeping the balance undisturbed" or "never destroying the equality" or "the use of axioms." But immediately thereafter, or within a few weeks, the shortcut of transposition is announced and thereafter, with very few exceptions, the boys and girls go through the motions of solving equations by mechanically following: "Put it on (or move it to) the other side and change its sign!" There is very rarely, indeed, any more mathematical thinking back of this than in "Put the block on (or move it to) the other end of the table and paint its face a different color."

"Put" and "move" are not mathematical terms; there are for procedure only (1) add, or (2) subtract, or (3) multiply, or (4) divide. To develop a foundation of understanding that will remain firm when needed to stand upon while dealing with formulas in physics, chemistry and higher mathematics, the *entire first* year of algebra is none too long a time to stay with the method that gives practice in developing clear *mathematical thinking.*

Several different good forms have been presented in magazine articles and in text books. For example:

$$\begin{array}{r} 2x-4=x+8 \\ +4 \quad +4 \\ \hline 2x \;=x+12 \\ -x \quad -x \\ \hline x \;=\; 12 \end{array}$$

And

$$\begin{array}{ll} & 2x-4=x+8 \\ \text{Add } 4 & 2x=x+12 \\ \text{Sub. } x \text{ or} & \\ \text{Add } (-x) & x=12 \end{array}$$

And

$$\begin{aligned} 2x-4&=x+8 \\ 2x-4+4&=x+8+4 \\ 2x&=x+12 \\ 2x-x&=x+12-x \\ x&=12 \end{aligned}$$

And several other slightly different forms.

After having used all three and after having watched results carefully and encouraged discussion in the classes, the second form seems to me somewhat superior, although each can be used in developing correct, clear thinking.

With the second form, after a few weeks, the majority will drop the *written* explanation at the left, a few will prefer to go on writing it. Very frequently indeed, throughout the entire year, students must be called upon to "speak out" the process of reasoning: Add 4 to both members of the equation. Subtract $x$ from, or add negative $x$ to, each member of the equation. Then *often*, the question must be asked: Why add that particular term and why is the equation not disturbed by that procedure?

Nothing has depth in consciousness without plenty of practice. This kind of work with the equation, never losing sight of what is really, mathematically being done is necessary *all* of the *first* year in algebra, if any large proportion of ninth or tenth graders are to have clear understanding required for success in physics, chemistry and higher mathematics.

Another shortcut which introduced too soon has caused confusion, is cross multiplication. Many try to use it where it does not apply, because they have not understood the principle upon which it is based and have not had enough practice in using that principle directly. The entire first year is not too long to "stick to meaningful drill on direct application of fundamental principles."

To clear an equation of fractions, multiply both members by the *same* factors and so keep the equality undisturbed. For example, in

$$\frac{1}{x-3}=\frac{2}{x+4}$$

$$\frac{1\cdot(x-3)(x+4)}{x-3}=\frac{2\cdot(x-3)(x+4)}{x+4}.$$

Multiply each member by $(x-3)$ $(x+4)$. Next, reduce the first member by dividing numerator and denominator by $(x-3)$ etc. Yes, it pays big dividends to take time for stating plainly these steps. A child will ask, "Shall I multiply or add?" Will the wise teacher reply in one word, or will time be taken to develop some reasoning there? If in arithmetic and algebra classes, the answers to problems are secured by mechanically, without understanding, following directions, most of the time might just as well be spent on crossword puzzles.

It is not just to get results which the teacher or answer book declares to be correct, that boys and girls should be "exposed" to arithmetic and algebra. It is not just to make passing grades or even A's and B's for any of a wide variety of reasons. Surely it should be that whether or not one goes to college, whether or not he ever uses the particular procedures as such, he shall have during the *study* of those subjects, grown as *much as possible* in the power of clear, logical thinking and shall have strengthened a number of desirable habits and attitudes.

# The Duke University Mathematics Institute

*By* FRANCES C. JOHNSON, *Senior High School, Oneonta, New York*
AND
FRANCES M. BURNS, *Senior High School, Oneida, New York*

HAVE you ever dreamed of an inspiring, relatively short summer course with no notes to take and with no examination to pass, and with a few delightful social events included? Then make your dream reality. Go to the Mathematics Institute next summer at Duke University.

"One of our greatest problems is to provide means for keeping the thousands of teachers who have received degrees in touch with the latest developments in education and in their special subjects." So states Dr. Carl Shuster, President of the National Council of Teachers of Mathematics. After visiting the 1946 session of the Mathematics Institute at Duke University he said, "It seems to me that the pioneer Institute that has been developed at Duke University is by far the best solution of the problem of providing maximum help for our mathematics teachers at a cost sufficiently low to enable them to avail themselves of the opportunity."

Some years ago, Professor W. W. Rankin of Duke University envisioned a program which would supplement the traditional college preparatory course and in-service training and thus meet the needs of many high school and college teachers and supervisors of mathematics. The first Institute (1941) ran for four days while the succeeding ones have averaged ten days in duration. As far as is known, this is the only mathematics institute held in the United States. That it fills a need is evident in the fact that the 1946 meetings were attended by 110 people from twenty-one states who travelled a total of 84,000 miles. It became international with the presence of two teachers from Canada and one from Puerto Rico. Classes, group meetings and lectures were not compulsory. There were no final examinations. No college credit was given. Because of a feeling that credit hours for the Institute would defeat its purpose, the program has been conducted free from all rewards except the intrinsic values gained by those in attendance. The only campus expense in 1946 was $3.00 university registration fee and a $3.00 charge per day for board and a single room or $2.50 per day for board and a double room in the university dormitories.

The work of the Institute centers around the Mathematics Laboratory which was begun three years ago by Professor Rankin. The purpose of the Mathematics Laboratory is fourfold:

1. To assemble in one place materials in mathematics and materials showing actual applications, and to make these materials easily available to teachers and students.
2. To encourage students and teachers to acquire broader concepts of the principles of mathematics and to learn the uses of mathematics in present-day civilization.
3. To promote better teaching in mathematics.
4. To build up a better understanding among educated people of the part mathematics plays in the world's work.

The Laboratory contains a large collection of present-day text books; a selection of books dealing with the applications of mathematics to science, engineering, industry, and commerce; books on the history of mathematics, the philosophy of mathematics, mathematical recreations and puzzles; curriculum studies; tests; charts; graphs; models and samples of pupil work. Recently there has been added a number of mathematical computing instruments, a chronometer, and replicas of seventeenth-century clocks. The estimated value of the Laboratory is $10,000. Teachers are encouraged to visit the Laboratory on week ends. It is expected that during the year work will begin on a set of

mural paintings known as "The Epic of Mathematics" which will set forth the story of mathematics and its uses over a period of 4,000 years. The paintings, which will become a part of the Mathematics Laboratory at Duke, are to be done by Miss Clare Leighton, an eminent English-American artist.

The 1946 program, the theme of which was "Theory and Practice Getting Better Acquainted," featured the lecture and the study group.

Lectures, which were scheduled each morning and evening, were given by key men from General Motors, International Business Machines Company, the United States Navy, DuPont Company, Sperry Gyroscope Company, Wright Automatic Machinery Company, and from the Clinton Laboratories of Oak Ridge, Tennessee. Talks were also given by educators from Duke, North Carolina State, and the University of North Carolina.

Typical of the study groups was that on Enrichment of Mathematics led by Miss Veryl Schult, Director of Secondary Mathematics, Washington, D. C., and assistant director of the Institute. Under her guidance the entire group participated in an exchange of ideas resulting in a wealth of suggestions and materials. The group was instructed also in curve stitching, in the making of nomographs, and in the construction of some simple models for plane geometry.

Other 1946 study groups were Consumer Mathematics, directed by Ruth F. Kimball of Washington, D. C.; Field Work in Mathematics under Professor Rankin; Applications of Mathematics to Science and Engineering given by Professor J. W. Cell, North Carolina State College.

A partial list of the study groups held during other sessions will indicate the variety of topics which were considered.

Mathematics in Aeronautics
Arithmetic in High School
Visual Aids in the Study of Mathematics
The Problem of Ninth and Tenth Year Mathematics
A Mathematics Laboratory
Mathematics for Students Not Going to College
Post-War Planning of Mathematics Programs
Aids in the Study of Geometry
Applications of Mathematics to Engineering
Traditional versus Unified Mathematics
The Teaching of Relationships

Prominent leaders who have contributed to the success of past sessions are Dr. William Betz of Rochester, New York; Dr. E. H. C. Hildebrandt of Northwestern University; Dr. W. S. Schlauch of New York University; Dr. William Brownell of Duke; Dr. Walter Carnahan of Purdue University; Professor W. J. Seeley of Duke; Professor Ruth Stokes of Winthrop College; Colonel F. M. Crist, United States Army; Dr. F. D. Murnaghan, Johns Hopkins University; Dr. Arthur S. Otis, World Book Company.

An important feature of the Institute is the booklet, *Mathematics Institute High Lights*, which is compiled during the sessions and is available at their close. The 1946 High Lights reached volume proportions in its 134 mimeographed pages. It contains a directory of those in attendance, complete texts of the lectures or summaries thereof, detailed reports of the study groups, notes on the Mathematics Laboratory, a list of materials available for the enrichment of mathematics with their sources and prices, and a sprinkling of the humor, art and poetry of the personnel. The summaries and reports are included to relieve one of the often distasteful and sometimes distracting task of note taking. One can sit back serenely in lecture or study group knowing that he will have a permanent (and legible) account of the proceedings in the High Lights.

An adequate concept of the Institute would not be connveyed unless mention were made of the delightful campus opportunities. The chapel services, the carillon concerts, the Sarah Duke Gardens, the hospitality of the university staff and the

social affairs held for Institute members enriched the days. Such affairs included a tea and a watermelon slicing at the home of Professor and Mrs. Rankin, dinners at the famous Carolina Inn, a reception at University House and a banquet honoring Vice-Admiral E. L. Cochrane, United States Navy.

A unanimous request was made for continuation of the Duke Institute. Suggestion was also made that the Duke Institute foster the idea in other sections of the country in order to serve a larger body of teachers.

At the close of the 1946 session a degree was conferred upon each teacher in attendance . . . the degree of Dynamic Teacher. The degree was granted by the Director by virtue of unstinted cooperation on the part of the candidates in adding to the knowledge and experience of others as well as themselves and of a willingness to accept the personal responsibility of revitalizing the teaching of mathematics.

After attendance at a Duke Institute no one doubts the validity of the theorem stated by Professor Rankin, "There is a simple and pleasant way to study mathematics."

The next Mathematics Institute will be held at Duke August 5–15, 1947. The theme will be "Mathematics At Work."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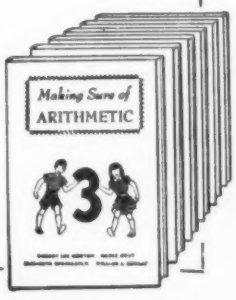


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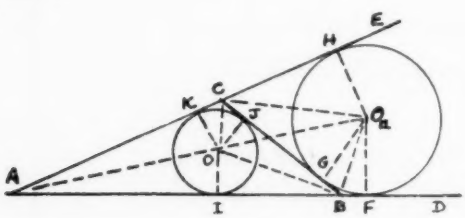
# Escribed Circles

*By* Joseph A. Nyberg
*Hyde Park High School, Chicago, Ill.*

Escribed circles have disappeared from modern texts in trigonometry in this country. Hence many teachers of high school mathematics have college credits in edution, calculus, statistics, and other subjects, but have never heard of escribed circles. This is unfortunate since a study of these circles offers an excellent opportunity to introduce more algebraic work in the trigonometry classes and to give a geometric interpretation of many of the terms that occur in certain formulas. The most thorough treatment of the topic is in Hall and Knight's Elementary Trigonometry (Macmillan Co. of London) but this is now a rare book.

An escribed circle is defined as a circle that is tangent to one side of a triangle and to the prolongations of the other sides. Hence every triangle has three escribed circles, whose radii are denoted by $r_a$, $r_b$, and $r_c$.




Some exercises about escribed circles appear in texts on plane geometry although the circles are not given that name. For example:

If $O_a$ is the intersection of the bisectors of the exterior angles $DBC$ and $ECB$ of $\triangle ABC$, then $AO_a$ bisects $\angle BAC$. An equivalent statement is: The bisector of an angle of a triangle and the bisectors of the opposite exterior angles are concurrent. This is a good exercise to use immediately after the class has proved the concurrence of the bisectors of the angles of a triangle.

If $O_aF$ is the perpendicular from $O_a$ to $AD$, then $AF$ equals half the perimeter of $\triangle ABC$ since

$$AF=\tfrac{1}{2}(AF+AH)$$
$$=\tfrac{1}{2}(AB+BG+AC+CG).$$

This is an exercise that every class in plane geometry should prove because $AF$ equals that famous letter $s$ which is used in Hero's formula for the area of a triangle and in many other formulas; and thus we have a geometric interpretation of that symbol.

Since $AF=s$ and $AB=c$, we have $BF=s-c$, and $BG=s-c$. Also, $CG=CB-BG=a-(s-c)=s-b$.

Let $I$, $J$, and $K$ be the points of tangency of the inscribed circle, and let $AI=x=AK$, $BI=y=BJ$, and $CJ=z=CK$. Then

$$x+y=c$$
$$y+z=a$$
$$z+x=b.$$

After the pupil has solved this set in any direct manner (a first solution should always be a direct one, and the clever solutions come later) attention may be called to the following method:

If the three equations are added and the sum divided by 2, then

$$x+y+z=\tfrac{1}{2}(a+b+c)=s.$$

If from this equation we next subtract in turn each of the equations of the set, then

$$z=s-c=CK=CJ$$
$$x=s-a=AI=AK$$
$$y=s-b=BI=BJ.$$

Note also that

$$IF=BI+BF=(s-b)+(s-c)=a$$
$$KH=KC+CH=(s-c)+(s-b)=a.$$

Hence $IF = BC = KH$. This is the familiar exercise of plane geometry: The external tangents and the segment of the common internal tangent between them are all equal.

The radius of an escribed circle can be found in the same way that the radius of an inscribed circle is found. To find $r$, we write

$$\triangle ABO + \triangle BCO + \triangle ACO = \triangle ABC$$

or

$$\tfrac{1}{2}cr + \tfrac{1}{2}ar + \tfrac{1}{2}cr = S \quad \text{or} \quad rs = S \quad \text{or} \quad r = \frac{S}{s}.$$

To find $r_a$ write:

$$\triangle ABO_a + \triangle ACO_a - \triangle BCO_a = \triangle ABC$$

or

$$\tfrac{1}{2}cr_a + \tfrac{1}{2}br_a - \tfrac{1}{2}ar_a = S$$

or

$$\tfrac{1}{2}r_a(c+b-a) = S.$$

But

$$c+b-a = 2(s-a);$$

hence

$$r_a = \frac{S}{s-a}.$$

Likewise,

$$r_b = \frac{S}{s-b}, \qquad r_c = \frac{S}{s-c}.$$

If the trigonometry class has previously proved $R = abc/4S$ for the radius of the circumscribed circle, then it can next prove the relation:

$$r_a + r_b + r_c - r = 4R$$

which is an interesting relation because it involves all the radii. Substitution gives:

$$\frac{S}{s-a} + \frac{S}{s-b} + \frac{S}{s-c} - \frac{S}{s} = \frac{abc}{S}.$$

Some pupils can see at once that each term should be multiplied by $S$, and the denominators will disappear. After doing this, some pupils laboriously perform all the multiplications while the more alert pupils group the terms two and two, factor and easily achieve their purpose. As part of the drill with logarithms, the pupils are asked to find the five radii for a triangle whose sides are given. Each pupil is given a different set of values for $a$, $b$, and $c$, so that he can experience the thrill that comes from making a long computation and then checking the results with the above formula. Four-place tables of logarithms are used.

The formula for $r_a$ can also be derived from the similar triangles $AIO$ and $AFO_a$.

$$\frac{IO}{AI} = \frac{FO_a}{AF} \quad \text{or} \quad \frac{r}{s-a} = \frac{r_a}{s}$$

or

$$r_a = \frac{rs}{s-a}.$$

Also

$$\tan \tfrac{1}{2}A = \frac{r}{s-a} = \frac{r_a}{s}.$$

By cyclic substitution,

$$\tan \tfrac{1}{2}B = \frac{r}{s-b} = \frac{r_b}{s}$$

$$\tan \tfrac{1}{2}C = \frac{r}{s-c} = \frac{r_c}{s}.$$

Then from $\triangle BFO_a$,

$$\tan\left[\tfrac{1}{2}(180-B)\right] = \cot \tfrac{1}{2}B = \frac{FO_a}{BF} = \frac{r_a}{s-c}.$$

Again, by cyclic substitution,

$$\cot \tfrac{1}{2}A = \frac{r_c}{s-b} \quad \text{and} \quad \cot \tfrac{1}{2}C = \frac{r_b}{s-a}.$$

Using $\triangle CHO_a$ in the same manner,

$$\cot \tfrac{1}{2}C = \frac{r_a}{s-b}, \qquad \cot \tfrac{1}{2}A = \frac{r_b}{s-c},$$

$$\cot \tfrac{1}{2}B = \frac{r_c}{s-a}.$$

Hence, from the various forms for tan $\frac{1}{2}A$ and cot $\frac{1}{2}A$,

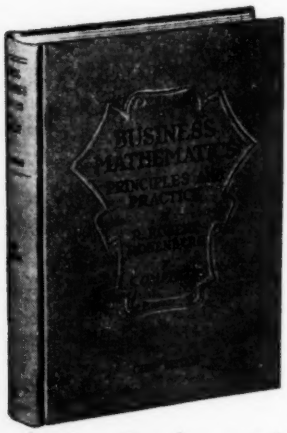
$$\frac{r}{s-a}=\frac{r_a}{s}=\frac{s-b}{r_c}=\frac{s-c}{r_b}.$$

Other relations can be found by using tan $\frac{1}{2}B$ and tan $\frac{1}{2}C$, or cyclic substitution may be applied to the above relations.

An interesting project for a mathematics club is that of drawing a figure showing all five circles and then finding the value of every segment and every angle in the figure in terms of $a$, $b$, $c$, $A$, $B$, and $C$. The triangle whose vertices are the centers of the escribed circles should also be investigated, and its sides, angles, area, inscribed and circumscribed and escribed circles found.

Other relations suitable for algebraic drill are:

1. If $r_a=r_b+r_c+r$ then $A=90°$.
2. $r_a=a\cos\frac{1}{2}B\cos\frac{1}{2}C\sec\frac{1}{2}A$
3. $r_a=4R\sin\frac{1}{2}A\cos\frac{1}{2}B\cos\frac{1}{2}C$
4. $\dfrac{r_a-r}{a}+\dfrac{r_b-r}{b}=\dfrac{c}{r_c}$
5. $rr_ar_br_c=S^2$
6. $rr_a\cot\frac{1}{2}A=S$
7. $r_ar_br_c=rs^2$
8. $r_ar_b+rr_c=ab$
9. $r_a+r_b=c\cot\frac{1}{2}C$
10. $r\cot\frac{1}{2}B\cot\frac{1}{2}C=r_a$
11. $(r_a-r)(r_b+r_c)=a^2$
12. $\dfrac{1}{r_a}+\dfrac{1}{r_b}+\dfrac{1}{r_c}=\dfrac{1}{r}$
13. $r_ar_b+r_br_c+r_cr_a=s^2$
14. $(r_a-r)(r_b-r)(r_c-r)=4Rr^2$
15. $\dfrac{b-c}{r_a}+\dfrac{c-a}{r_b}+\dfrac{a-b}{r_c}=0.$

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# Teaching Signs in Fractions

*By* Elbert Fulkerson
*Southern Illinois Normal University, Carbondale, Illinois*

It has been the writer's observation through many years of teaching mathematics in the secondary schools and in the supervision of student teaching in this subject, that a relatively large number of students who have studied considerable mathematics in high school, and far too many college students who are preparing to become teachers of mathematics, are not thoroughly grounded in the principles underlying the changing of signs in fractions. It will be, therefore, the purpose of this article to outline a plan which the author has found to work quite satisfactorily in the teaching of the signs in fractions, with the hope that such a plan might be used advantageously by beginning teachers in the field of mathematics.

The beginning algebra class is first taught all of the ordinary processes concerning the manipulation of fractions without any mentioning of signs in fractions. Then to arouse interest in the use of the signs, the class is asked to reduce to lowest terms some fraction in which the factors of the numerator and the denominator will be oppositely arranged. For example, the fraction

$$\frac{x^2-4x+4}{4-x^2}$$

will be written on the board with the direction that the class reduce this fraction to lowest terms. Now the class has already been taught that a fraction is reduced to lowest terms by factoring both numerator and denominator and cancelling out like factors. The students will soon see that by factoring this example they will have the fraction

$$\frac{(x-2)(x-2)}{(2-x)(2+x)},$$

which appears to have no like factors common to the numerator and denominator. They will also readily discern that the factor, $2-x$, in the denominator would be like the factor, $x-2$, in the numerator if the sign of each term in $2-x$ could be changed, thus making it $x-2$. Hence, the original fraction could be reduced to lower terms if one were permitted to change signs. At this point the teacher brings up the fact that signs in fractions may always be changed, provided one does not violate certain fundamental principles. The students are now extremely interested in discovering just what principles work here and how they can be applied to the solution of this problem.

The teacher then emphasizes the fact that the changing of signs in fractions all depends upon that fundamental principle that both the numerator and denominator of a fraction may be multiplied or divided by the same quantity without changing the value of the fraction. If any members of the class have difficulty in understanding what this principle means, it can be brought out that this was precisely the thing which was done when they changed fractions to like fractions or reduced fractions to lowest terms in arithmetic processes. Now the teacher proceeds to show how this principle can be applied.

Then consider the fraction $a/b$. Every fraction must always have three signs—the sign before the numerator, the sign before the denominator, and the sign before the fraction. In this fraction all three signs are positive, but it can be demonstrated that the fraction can be written as $-a/-b$, $-(-a/b)$, $-(a/-b)$. First it is to be proved that

$$\frac{a}{b}=\frac{-a}{-b}.$$

Now

$$\frac{a}{b}=\frac{-1(a)}{-1(b)}.$$

Both the numerator and denominator of a fraction may be multiplied by the same quantity (here $-1$) without changing the value of the fraction. By actually performing the indicated multiplication, the fraction $a/b$ becomes the fraction $-a/-b$. From this transformation it can be concluded that the signs of both the numerator and denominator can be changed without changing the value of the fraction. At this point the teacher must exercise particular care to prevent the class from jumping to the conclusion that the sign of the numerator or denominator is the sign of the first term in each. Just so long as the numerator and denominator are monomials the students will have little difficulty in recognizing the signs of either. However, if either one should be a polynomial, then the students often mistake the sign of the first term for the sign of the whole numerator or denominator as the case may be. For example, if the student found it necessary to change the signs of the numerator and denominator in the fraction

$$\frac{x^2-4x+4}{4-x^2},$$

he must be taught that this would mean the changing of the sign of every term in both the numerator and denominator, thus obtaining the fraction

$$\frac{-x^2+4x-4}{-4-x^2}.$$

Next it is to be shown that

$$\frac{a}{b}=-\frac{-a}{b}.$$

Now the fraction

$$\frac{a}{b}=\frac{(-1)(-1)a}{b}.$$

Here the numerator of the fraction is to be multiplied by $-1$ used twice as a factor. Obviously this is the same as multiplying the numerator once by $+1$, and does not, therefore, change the value of the numerator and hence not the value of the whole fraction. It is desired, however, to make the sign of the numerator negative. Hence the numerator will actually be multiplied by one $-1$, and the other $-1$ will be held as an indicated multiplication in the numerator. Then $a/b$ becomes

$$\frac{(-1)(-a)}{b}.$$

But to indicate that the numerator of a fraction is to be multiplied by any quantity is the same as indicating that the whole fraction is to be multiplied by that quantity. If the students have difficulty in interpreting this statement, let the teacher ask them how one would multiply the fraction 2/3 by the whole number 5. They will easily see that this is done by multiplying just the numerator by the 5. That is,

$$\frac{5(2)}{3}$$

is the same as

$$5\left(\frac{2}{3}\right).$$

In the same manner

$$\frac{(-1)(-a)}{b}$$

can be written as

$$(-1)\frac{-a}{b}.$$

But here the numerical coefficient of the fraction $-a/b$ is $-1$, from which the 1 may be omitted in actual writing. Then the fraction

$$(-1)\frac{-a}{b}$$

becomes

$$-\frac{-a}{b}.$$

From this it can be concluded that the sign of the numerator of a fraction may be changed by changing the sign before the fraction. Here again, it should be emphasized that the sign of the numerator means the sign of every term of the numerator in case it is a polynomial expression.

Finally it is to be proved that

$$\frac{a}{b}=-\frac{a}{-b}.$$

Again, the fraction

$$\frac{a}{b}=\frac{(-1)a}{(-1)b}.$$

Multiplying both numerator and denominator by $-1$ does not change the value of the fraction. In this case, however, it is desired to keep the numerator positive. Hence, instead of actually multiplying the numerator by the $-1$ the fraction is multiplied by this $-1$. Now

$$\frac{(-1)a}{(-1)b}$$

can be written as

$$(-1)\frac{a}{-b}$$

and then by omitting the numerical coefficient of 1 before the fraction and performing the indicated multiplication in the denominator, this fraction becomes $a/-b$. Thus, the sign of the denominator of a fraction may be changed by changing the sign before the fraction.

At this point the teacher asks the class to apply the information toward the solution of the original example. The students will now readily see that, if the denominator of the fraction

$$\frac{x^2-4x+4}{4-x^2}$$

were changed to $x^2-4$ before factoring, the factor $x-2$ in the denominator would be identical with the factor in the numerator, and cancellation could be performed. They will further see that this denominator can be so changed by changing the sign of every term it contains. But this will necessitate a changing of the sign before the fraction. The steps might be written on the board in the following form:

$$\begin{aligned}\frac{x^2-4x+4}{4-x^2}&=-\frac{x^2-4x+4}{x^2-4}\\&=-\frac{(x-2)(x-2)}{(x-2)(x+2)}\\&=-\frac{(x-2)(x-2)}{(x-2)(x+2)}\\&=-\frac{(x-2)}{(x+2)}.\end{aligned}$$

Following this solution, the students should be given several examples requiring an application of the principles in order that the knowledge may be fixed while the interest is keen.

In many manipulations the student may encounter the fraction with its numerator and denominator already in the factored form, but not properly arranged for easy simplification. Then he might think it necessary, before applying the principles outlined above, to find the product of the factors, rearrange the terms, and again factor, all of which may result in an involved complicated procedure, especially if the numerator or denominator should contain several factors. To obviate this difficulty, the students should be taught the application of the principles when the numerator and denominator are in the factored form. For example, let it be required to reduce the fraction

$$\frac{(3-x)(x-5)(1-x)}{(x-2)(x-3)(x-1)}$$

to lowest terms. Here the factors $3-x$ and $1-x$ in the numerator can be made identical to the factors $x-3$ and $x-1$ in the denominator by changing the sign of every term in each of the two factors. This can be done by multiplying each factor by $-1$. Here it should be pointed out that every

time a factor of an indicated product is multiplied by any quantity, the whole product is multiplied by that same quantity. If the students do not understand this principle, a simple numerical illustration may be used to clarify the idea. For example, (2) (3) (5) = 30. Now if the first factor is multiplied by 2, the example becomes (4) (3) (5), which gives 60, or a quantity twice as large as the original product. Other factors of the example might be treated similarly with the same results. Now when the factor $3-x$ is multiplied by $-1$, this is equivalent to multiplying the whole numerator of the fraction by $-1$. Again, when the factor $1-x$ is multiplied by $-1$, the whole numerator is once more multiplied by $-1$. But by multiplying any quantity twice by $-1$ is equivalent to multiplying that quantity by $+1$, which obviously does not change the value of the original quantity. Thus the fraction

$$\frac{(3-x)(x-5)(1-x)}{(x-2)(x-3)(x-1)}$$

becomes

$$\frac{(x-3)(x-5)(x-1)}{(x-2)(x-3)(x-1)}.$$

By cancelling like factors in the numerator and denominator, this fraction reduces to

$$\frac{x-5}{x-2}.$$

Now, if other examples are used, where it is necessary to change the signs of the terms of one, three, five, or any odd number of factors, in either the numerator or denominator, it can be readily shown that this would be equivalent to multiplying the whole numerator or denominator by $-1$, which in turn would actually be the same as changing the sign of every term of the numerator or denominator. In such a case, then the sign of the fraction would need to be changed in order to maintain the original value. From these and similar examples brought to the attention of the students, the procedure for changing the signs of factors in fractions may be summarized in the following general principles: The signs of all the terms in an even number of factors of the numerator or denominator or of both, may be changed without changing the sign before the fraction. The signs of all the terms of an odd number of factors of either the numerator or denominator may be changed by changing the sign before the fraction. The signs of all the terms of an odd number of factors of both the numerator and denominator may be changed without changing the sign before the fraction.

While some students prefer to memorize the specific principles and use these thereafter as needed, without any further thought of rationalization, the author prefers to teach the students to make use of the fundamental principle that both terms of a fraction may be multiplied or divided by the same quantity without changing its value, when they need to change signs of the numerator or denominator, or of factors in the numerator or denominator. This multiplier or divisor, as the case may be, is $-1$. Finally, he likes to call the attention of the class to the fact that every fraction has three signs, any two of which may be changed without changing the value of the fraction.

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**On to Atlantic City!**

See the January issue of THE MATHEMATICS TEACHER for the complete program of The Annual Meeting of the National Council of Teachers of Mathematics at Chalfonte-Haddon Hall

# Do First Graders Retain Number Concepts Learned in Kindergarten?

*By* Sina Mott and Mary Elizabeth Martin
*Southern Illinois Normal University, Carbondale, Ill.*

An investigation to determine the amount of number concepts retained by children from one grade to the next was made of groups of children attending the Laboratory School of Southern Illinois Normal University. The first two groups were studied during the year of 1944–45 by Dr. Sina M. Mott. At that time, the children were enrolled in Kindergarten. The second study was made with the same children upon entering the first grade the following fall of 1945.

The procedure used in testing the Kindergarteners was as follows. The children were interviewed separately so that the remarks of one might not influence the response of another. The teacher and child sat at a small table which contained fifteen small colored cubes, a toy truck containing twenty-eight cubes, a working sheet, pencil and paper. In interviewing each child, he was asked to count by rote, count objects, repeat a series of four numbers, pick a designated number of cubes from a pile of fifteen cubes, and repeat a series of five numbers. Table I contains the responses given to questions: (1) counting by rote, (2) counting objects, (3) repeating four numbers, (4) repeating five numbers, (5) picking the numbers 3, 9, 5 and 7 from a pile of fifteen blocks.

In summarizing the results of the first test as recorded in Table I, 96% counted by rote beyond ten, 82% to fifteen, 50% beyond twenty, and 22% to one hundred.

In object counting twelve of the sixteen children counted by objects beyond their rote counting. 93% counted beyond ten, 87% beyond fifteen, 81% beyond twenty and 50% beyond twenty-eight.

In repeating numbers, 82% repeated all three sets of the group of four numbers correctly. 33% could repeat the sets of five numbers correctly.

The following table shows the per cent of those counting aloud and those counting

Table I

*Counting and Repeating Numbers, and Number Concepts*
*Kindergarten*

| Pupil | Age mo. | Counting | | Repeating Numbers | | | | | | Concepts | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Rote | Object | 4729 | 3852 | 7261 | 31859 | 48372 | 96183 | 3 | 9 | 5 | 7 |
| B.C. | 73 | 19 | 19 | o.k. | o.k. | 7621 | 3895 | 47 | 9386 | CE | CL | CE | CL |
| S.S. | 70 | 100 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | CE | CE | CE | CE |
| L.B. | 68 | 12 | 29 | o.k. | o.k. | o.k. | 3189 | 3872 | o.k. | CE | CL | CL | CL |
| D.L. | 67 | 100 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | CE | CE | CE | CE |
| L.C. | 66 | 20 | 26 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | CE | CL | CL | CL |
| L.F. | 65 | 19 | 28 | o.k. | o.k. | o.k. | 3859 | o.k. | o.k. | CE | CE | CE | CE |
| P.W. | 65 | 23 | 25 | o.k. | o.k. | o.k. | o.k. | o.k. | 96813 | CL | CL | CL | CL |
| J.O. | 64 | 13 | 25 | o.k. | o.k. | o.k. | 3589 | 4732 | 961 | CL | CL | CL | CL |
| F.H. | 64 | 30 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | 9683 | CL | CL | CL | CL |
| M.H. | 62 | 29 | 29 | o.k. | 3 | o.k. | 3 | 4872 | 978 | CL | CL | CL | CL |
| F.S. | 61 | 15 | 28 | o.k. | o.k. | o.k. | o.k. | 4872 | 9613 | CE | CE | CE | CE |
| E.L. | 62 | 100 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | CE | CL | CL | CL |
| D.H. | 72 | 20 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | CE | CL | CE | CL |
| F.C. | 71 | 14 | 14 | o.k. | o.k. | o.k. | 1859 | 8172 | 96183 | CE | CL | CL | CL |
| H.B. | 72 | 100 | | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | CL | CL | CL | CL |
| M.R. | 65 | 100 by 2 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | CE | CE | CE | CE |

CE, counting with eyes; CL, counting out loud.

with their eyes when being tested for the number concepts of three, nine, five and seven.

| | 3 | 9 | 5 | 7 |
|---|---|---|---|---|
| Counting out loud | 31% | 68% | 56% | 68% |
| Counting with eyes | 68% | 31% | 43% | 31% |

The second test was made with the same group of children after they entered the first grade. A similar procedure of testing was followed. The child was asked to count by rote, count objects, repeat a series of four numbers, pick a designated number of blocks from a pile of fifteen blocks, and repeat a series of five numbers. Table II contains the responses given to the questions: (1) counting by rote, (2) counting objects, (3) repeating four numbers, (5) picking the numbers 3, 9, 5, and 7 from a pile of blocks.

In summarizing the results of the second test as recorded in Table II, 100% counted by rote beyond ten, 93% to fifteen, 81% beyond twenty, and 12% to one hundred.

In object counting, thirteen of the sixteen children counted by objects beyond their rote counting. 100% counted objects beyond ten, 93% beyond fifteen, 75% beyond twenty, and 68% beyond twenty-eight.

In repeating numbers, 93% repeated all three sets of the group of four numbers correctly. 37% could repeat the sets of five numbers correctly.

The following table shows the per cent for those counting aloud and those counting with their eyes.

| | 3 | 9 | 5 | 7 |
|---|---|---|---|---|
| Counting out loud | 0 | 62% | 0 | 62% |
| Counting with eyes | 100% | 37% | 100% | 37% |

## Summary

By comparing the per cents in the following tables, one can readily see that, with the exception of counting by rote to 100, children carry over into the first grade after a lapse of approximately three months, the number experiences which they have learned in kindergarten. The explanation for not retaining the ability to count by rote to 100 is possibly due to the fact that children have little use for numbers of this quantity and are therefore meaningless to them. Hence, they do not

Table II

*Counting and Repeating Numbers, and Number Concepts*
*1st Grade*

| Pupil | Age mo. | Counting | | Repeating Numbers | | | | | | Concepts | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Rote | Object | 4729 | 3852 | 7261 | 31859 | 48372 | 96183 | 3 | 9 | 5 | 7 |
| B.C. | 71 | 15 | 27 | o.k. | o.k. | o.k. | 3859 | o.k. | 9863 | CE | CL | CE | CL |
| S.S. | 78 | 100 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | CE | CL | CE | CE |
| L.B. | 71 | 14 | 16 | o.k. | o.k. | o.k. | 3859 | 38472 | 396 | CE | CL | CE | CL |
| D.L. | 69 | 39 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | CE | CL | CE | CL |
| L.C. | 70 | 26 | 12 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | CE | CE | CE | CE |
| L.F. | 73 | 30 | 28 | o.k. | o.k. | o.k. | 38159 | o.k. | o.k. | CE | CL | CE | CL |
| P.W. | 76 | 23 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | CE | CL | CE | CL |
| J.O. | 72 | 18 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | CE | CL | CE | CL |
| F.H. | 72 | 29 | 28 | o.k. | o.k. | o.k. | o.k. | 4872 | o.k. | CE | CE | CE | CL |
| M.H. | 75 | 49 | 16 | o.k. | o.k. | o.k. | 95183 | 46872 | 98683 | CE | CL | CE | CL |
| F.S. | 68 | 25 | 28 | 7429 | o.k. | 1267 | 38159 | o.k. | o.k. | CE | CE | CE | CL |
| E.L. | 76 | 28 | 28 | o.k. | o.k. | o.k. | 38159 | o.k. | o.k. | CE | CE | CE | CE |
| D.H. | 78 | 79 | 28 | o.k. | o.k. | o.k. | 38159 | o.k. | o.k. | CE | CE | CE | CE |
| F.C. | 72 | 39 | 17 | o.k. | o.k. | o.k. | o.k. | 38472 | o.k. | CE | CL | CE | CL |
| H.B. | 78 | 49 | 28 | o.k. | o.k. | o.k. | 3189 | o.k. | 91683 | CE | CL | CE | CE |
| M.R. | 73 | 100 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | CE | CL | CE | CE |

CE, counting with eyes; CL, counting out loud.

TABLE III

*Comparison of the Tests in Counting and Repeating Numbers*

| Pupil | Rote | | Object | | 4729 | | 3852 | | 7261 | | 31859 | | 48372 | | 96183 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Kin. | 1st | Kin. | 1st | Kin. | 1st | Kin. | 1st | Kin. | 1st | Kin. | 1st | Kin. | 1st | Kin. | 1st |
| B.C. | 19 | 15 | 19 | 27 | o.k. | o.k. | o.k. | o.k. | 7621 | o.k. | 3895 | 3859 | 47 | o.k. | 9683 | 9863 |
| S.S. | 100 | 100 | 28 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. |
| L.B. | 12 | 14 | 29 | 16 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | 3189 | 3859 | 3872 | 38472 | o.k. | 396 |
| D.L. | 100 | 39 | 28 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. |
| L.C. | 20 | 26 | 26 | 12 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. |
| L.F. | 19 | 30 | 28 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | 3859 | 38159 | o.k. | o.k. | o.k. | o.k. |
| P.W. | 23 | 23 | 25 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | 96813 | o.k. |
| J.O. | 13 | 18 | 25 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | 3589 | o.k. | 4732 | o.k. | 961 | o.k. |
| F.H. | 30 | 29 | 28 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | 4872 | 9683 | o.k. |
| M.H. | 29 | 49 | 29 | 16 | o.k. | o.k. | 3 | o.k. | o.k. | o.k. | 3 | 95183 | 4872 | 46872 | 978 | 986 |
| F.S. | 15 | 25 | 28 | 28 | o.k. | 7429 | o.k. | o.k. | o.k. | 1267 | o.k. | 38159 | 4872 | o.k. | 9613 | o.k. |
| E.L. | 100 | 28 | 28 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | 38159 | o.k. | o.k. | o.k. | o.k. |
| D.H. | 20 | 79 | 28 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | 3259 | o.k. | 3872 | o.k. | 3169 |
| F.C. | 14 | 39 | 14 | 17 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | 1859 | o.k. | 8172 | 38472 | 96183 | o.k. |
| H.B. | 100 | 49 | | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | 3189 | o.k. | o.k. | o.k. | 91683 |
| M.R. | 100 by 2 | 100 | 28 | 28 | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. | o.k. |

TABLE IV

*Comparison of the Tests in Number Concepts*

| Pupil | 3 | | 9 | | 5 | | 7 | |
|---|---|---|---|---|---|---|---|---|
| | Kin. | 1st | Kin. | 1st | Kin. | 1st | Kin. | 1st |
| B.C. | CE | CE | CL | CL | CE | CE | CL | CL |
| S.S. | CE | CE | CE | CE | CE | CE | CE | CE |
| L.B. | CE | CE | CL | CL | CL | CE | CL | CL |
| D.L. | CE | CE | CE | CL | CE | CE | CE | CL |
| L.C. | CE | CE | CL | CE | CL | CE | CL | CE |
| L.F. | CE | CE | CE | CL | CE | CE | CE | CL |
| P.W. | CL | CE | CL | CL | CL | CE | CL | CL |
| J.O. | CL | CE | CL | CL | CL | CE | CL | CL |
| F.H. | CL | CE | CL | CE | CL | CE | CL | CL |
| M.H. | CL | CE | CL | CL | CL | CE | CL | CL |
| F.S. | CE | CE | CE | CE | CE | CE | CE | CL |
| E.L. | CE | CE | CL | CE | CL | CE | CL | CE |
| D.H. | CE | CE | CL | CE | CE | CE | CL | CE |
| F.C. | CE | CE | CL | CL | CL | CE | CL | CL |
| H.B. | CL | CE | CL | CL | CL | CE | CL | CE |
| M.R. | CE | CE | CE | CL | CE | CE | CE | CE |

CE, counting with eyes; CL, counting out loud.

TABLE V

| Rote Counting | | |
|---|---|---|
| | Kindergarten | 1st grade |
| Beyond 10 | 96% | 100% |
| Beyond 15 | 82% | 93% |
| Beyond 20 | 50% | 81% |
| Count to 100 | 22% | 12% |

TABLE VI

*Object Counting. 12 of the 16 children in Kindergarten, as compared with 13 of the 16 children in the first grade counted by objects beyond their rote counting.*

| | Kindergarten | 1st grade |
|---|---|---|
| Beyond 10 | 93% | 100% |
| Beyond 15 | 87% | 93% |
| Beyond 20 | 81% | 75% |
| Beyond 28 | 50% | 68% |

retain them. The obvious reason that children do retain the other number concepts and experiences from one grade to the next is the fact that they are constantly aware of the need for counting objects and repeating numbers, and are using them frequently.

TABLE VII

*Repeating Numbers*

| | Kindergarten | 1st grade |
|---|---|---|
| Repeating 4 numbers | 82% | 93% |
| Repeating 5 numbers | 33% | 37% |

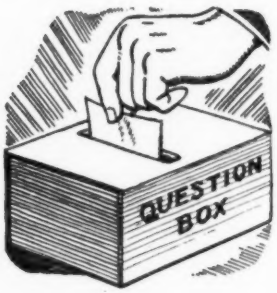
TABLE VIII

*Number Concepts*

| | Kin. | 1st | Kin. | 1st | Kin. | 1st | Kin. | 1st |
|---|---|---|---|---|---|---|---|---|
| | 3 | | 9 | | 5 | | 7 | |
| Counting aloud | 31% | 0 | 68% | 62% | 56% | 0 | 68% | 62% |
| Counting with eyes | 68% | 100% | 31% | 37% | 43% | 100% | 31% | 37% |

*A Question-Box Report on*

ARITHMETIC
FOR
YOUNG AMERICA

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# Ninth Year Mathematics*

*By* WALTER H. CARNAHAN
*Purdue University, Lafayette, Ind.*

MANY a person whose experiences in high school mathematics, either as student or teacher, date back to the early years of this century longs for those good old days when ninth year mathematics meant the same thing to every one in all schools, namely algebra through quadratics. In those days, there were no doubts about the value of alegbra for every student in the ninth year, no questions raised about adjustment of subject matter to the needs and interests of students, and no suggestions for changed organization of subject matter in the interest of more effective learning.

If mathematics was taught in sequential courses, that was as it should be. If there were "water tight compartments," they were good and they served a useful educational purpose, so why break the barriers? Moreover, no one had heard of water tight compartments, and why coin troublesome expressions?

But in those early days of the twentieth century not everything mathematical was as it should be, neither in America, England, Germany nor France. Observant men realized this, and thoughtful men believed reforms were in order. Perry in England, Klein in Germany, Tannery in France and Moore in America simultaneously raised their voices in demand for improvement. These men pointed out that learning of mathematics was retarded because only a limited part of its power could be brought to bear in the attack on the difficulties encountered in any given course, forgetting was rapid because after a subject had been learned and a course had been passed its information and skills were seldom again brought out for use; effectiveness of application to problem situations encountered outside the class room was disappointing because such problems cannot usually be classified as arithmetic, or algebra, or geometry, or trigonometry, and students who have grown accustomed to problems so classified are at a disadvantage when classification is not possible.

Moore, Klein, Perry, Tannery and a few others were prophets. In each country there sprang up their followers who became the crusaders. Among these crusaders in America were Myers, Breslich, Reeve, Schorling, Clark and many others. Realizing that the ideas of the prophets had to be made very, very concrete if they were to get down where the teaching and learning are done, the crusaders began to present their ideas in methods courses, in magazine articles, in speeches, and in textbooks. Problems of class organization, pupil selection, teaching techniques, crediting, sequential organization and other problems had to be solved. School administrators, teachers, pupils and parents had to be evangelized and instructed.

There was lack of agreement among the leaders on many fundamental points. For example, what would the new mathematics be called? Fusion mathematics? Integrated mathematics? Cooperative mathematics? Coordinated mathematics? Unified mathematics? Composite mathematics? Correlated mathematics? General mathematics? Since the proponents of the new organization were not in agreement as to the name, it was inevitable that opponents should take full advantage of this minor confusion. *Fusion mathematics* was often called *confusion mathematics; integrated mathematics* became *aggravated mathematics; cooperative mathematics* became *non-operative mathematics;* and one teacher of army experience and vocabulary called it *slumgullion,* a soldier's name for food fit for swine. We may refer to this as

* A paper read at the Mathematics Section of The Northwestern Indiana State Teachers Association, October 24, 1946.

the *name calling period* of mathematics reorganization in America. This period has about passed. It passed with the passing of the multiplicity of names. I think it safe to say that there will be teachers who read this article who have never heard the names fusion mathematics, or coordinated mathematics, or composite mathematics. And some have heard no name other than general mathematics. That, I think, is evidence of some considerable progress. To have agreed on a name is in itself no small achievement, and to have reached other corollary agreements is a still greater achievement.

Let us examine some of these corollary agreements. It seems to be agreed that integration of mathematics should be practiced only where it is a natural thing to do; no "shot gun weddings" of topics in mathematics. There was a time when two remotely related or wholly unrelated topics would be thrown together in a text as if by such an act of violence they could be unified. That time has passed or is rapidly passing. Writers of textbooks generally realize that mere proximity, or juxtoposition, or emulsion of topics in a book do not satisfy any reasonable criterion of favorable learning conditions. In preparation for the frequent transitions from one subject to another, from simple algebra to intuitive geometry, or from intuitive geometry to arithmetic applications, the way must be made very smooth and the curves in the road must be very gentle. Not only that, but the destination to be reached by following the road must be constantly in view. The destination may be functional competence, or mathematical power, or the ability to think clearly, or cultural development, or intellectual enjoyment, or a combination of all of these. Whatever it may be, it must be carefully kept in view and in nearly all modern texts for ninth year mathematics it is so kept in sight. That indicates distinct progress.

Another of the agreements corollary to the agreement on a name is what subjects belong in ninth year mathematics. This agreement is so universal that there is no hesitancy in naming the subjects. They are: (1) Algebra, (2) arithmetic, (3) informal geometry, (4) numerical trigonometry. The above statements can be easily verified. Take up a dozen books written within the last fifteen years for ninth year mathematics, books that are reasonably widely used, and eleven of the dozen will include treatment of the four subjects listed above. As a basis for comparison, examine a dozen books in wide use twenty-five years ago; no such agreement will be found. Such present agreement on subjects marks distinct progress.

I digress to tell a true story. A few weeks ago I had a long discussion with a man who has leadership responsibility for a state program in the social studies. He listed a number of points on which he was encountering great difficulty in developing a program that would be generally acceptable. If a program were acceptable to one group of social studies teachers, another group would attack it vigorously, even acrimoniously. "Why," he asked, "should this be the situation in social studies when it is not true to the same degree in mathematics?" It seems to me that the difference is merely one of time. My guess is that social studies teachers are now passing through the period in which teachers of mathematics found themselves a quarter of a century ago; the period of debate, experimentation, evaluation, and perhaps a little harmless name calling. It is the good old American way.

Having pointed out certain agreements to which we have arrived as a result of some years of mathematical experience, I invite your attention to some matters on which agreement has not yet been reached. First, there is the question of a name, a question greatly reduced in magnitude from the early nineteen hundreds, as has been seen. What should we call the course in ninth year mathematics. Algebra or general mathematics? (Now, as you read this, please sit quietly in your chair and keep your blood pressure normal. No

shouting and no name calling, please. I seem to hear someone shout "Bootlegger! Carrying trigonometry in an algebraic suitcase!" and someone else seems to be shouting "Thief!" Painting a new name on an old subject and selling it as "brand new!") The matter of a name for ninth year mathematics would be somewhat simplified if colleges would change the entrance requirement from "1 unit of algebra" to "1 unit of mathematics including algebra through quadratics." With such a change, school administrators inclined to literalism would not feel compelled to name a course and use a book with the algebra label. The administrator who is not so literal recognizes no such problem. He gives whatever course seems best for ninth year students, selects the book that contains what he wants to have taught and records one unit of credit. When the question of college certification arises, he consults the college catalog to see what label the school prefers and applies it with a clear conscience to the earned credit, provided the course as taught in his school includes the topics usually associated with that label.

Another matter on which general agreement remains to be reached is the matter of geometry in ninth year mathematics. The specific questions remaining to be answered are, for what purpose, how much, and what kind?

Why is geometry included in ninth year mathematics? Is it justified on the basis of making mensuration formulas seem reasonable, or for rationalizing quantitative relations to be used in problems and possible later practical situations? Is it justified on the basis of the value of constructing and observing detail characteristics of geometric figures? Is the purpose to establish selected propositions informally preliminary to future demonstrative treatment? Is the purpose to give early experience with postulational reasoning?

The purpose of this article is to summarize, to report, to inventory, not to present arguments nor to urge conclusions. Consequently I resist the temptation to try to make converts to my own point of view regarding the justification of geometry in ninth year mathematics. It seems safe to say, however, that at present the tendency is to reduce greatly the amount of geometry included in ninth year mathematics. The number and prominence of axioms and postulates tends to be reduced; the number of propositions introduced is decreasing; proofs become less formal; the number of constructions taught tends to be reduced. These conclusions may be questioned; they are based on comparison of books published over a period of twenty-five years, not alone on those published during the last ten years or so, and the comparison seems to justify the statements made.

There are still persons who question the introduction of any geometry into ninth year mathematics and look on this as something wholly new. It is new only in its placement in the ninth year. Formerly it was quite generally introduced into seventh or eighth year mathematics. An examination of arithmetic texts published fifty or more years ago shows that mensuration was treated in nearly all books written for use in seventh and eighth years. The purpose then was not to teach geometry as such, but to rationalize rules for perimeter, area and volume. However, so far as the facts learned or their rationalization are concerned, the purpose might as well have been the teaching of a limited amount of informal geometry. (Question: Did the introduction of mensuration make these books texts in general mathematics?)

Still another point on which general agreement remains to be reached in the organization of ninth year mathematics is that of arithmetic. Specifically, the agreement to be sought is in regard to the purpose of arithmetic in the course; that there should be some arithmetic is quite generally accepted. Is the purpose to introduce topics too difficult for previous mastery, or topics for which there was not sufficient time, or topics too far removed from the

social experiences of the students? Or is the purpose to maintain and sharpen skills previously acquired and to develop fuller understanding of processes? It seems that the tendency now is to introduce into the ninth year arithmetic for maintenance and better understanding rather than for treatment of new topics. Again, this conclusion is subject to challenge, but examination of the published texts prompts this statement.

In this discussion, the writer has in mind the mathematical needs of non-special groups of high school students, not vocational students, or retarded students, or wholly disinterested students. Such students present special problems. Shall they be given applied mathematics, shop arithmetic, arithmetic for nurses, review arithmetic, social arithmetic, business arithmetic? These are important problems, but no attempt at their solution is here indicated. What we have been discussing is mathematics for those who have fair ability, moderate interest, willingness to be guided, a reasonable probability of continuing the study of mathematics beyond the required minimum, or the possibility of the use of mathematics in science or technology.

However, for these "ordinary pupils" (if I may be permitted to use this expression without defining it) mathematics presents itself in a great variety of forms. Examination of some of the text books offered for ninth year use impresses one with this variety. It would not be possible to examine their contents in this discussion, but even a partial list of the titles will suggest the diversity of offerings. Here are a few: *Mathematics in Life, Vital Mathematics, Purposeful Mathematics, Living Mathematics, Mathematics in Daily Use, Mathematics in Daily Life, New Applied Mathematics, Mathematics at Work, Mathematics for Every Day Affairs, Essential Mathematics, Home and Job Mathematics, Useful Mathematics, Everyday Mathmatics, Everyday Junior Mathematics, New Practical Mathematics, Mathematics Everyday, Mathematics for Everyone.*

If it be said that many of these books were never intended by authors nor publishers for use with "ordinary ninth year pupils," I respectfully reply that they are in fact so used in some high schools, and some of the above titles are included in at least one circulated list of general mathematics texts, which implies that those who compiled the list believe that these books qualify as text materials for courses to be given on the same basis as ninth year algebra.

Most state departments of education publish courses of study in mathematics and prescribe minimum mathematical requirements for high school graduation. Many such courses and prescribed requirements list general mathematics as one of the subjects meeting mathematical requirements, but not often is general mathematics carefully defined in state administrative handbooks and courses of study. If the states adopt text books for high school use, the pattern for courses is thus laid out, but where local schools are free to adopt texts without any limitations, the pattern for ninth year mathematics is likely to vary widely from school to school. One remedy for this is for state education departments to define carefully what is intended by their use of the term *general* mathematics. Another remedy is time; I have indicated the effect of time as a stabilizing influence in course content and in use of a name for the new ninth year mathematics. Without doubt more time will mean still further stabilization, at least until accumulated educational problems call for other radical mathematical reorganization such as that of forty years ago.

What pattern will ninth year mathematics follow during the years immediately ahead? To answer that question is to pose as a prophet, and prophesies have a way of returning to embarrass those who utter them. Nevertheless, the signs seem to justify certain general predictions: (1)

Ninth year mathematics will continue to consist of algebra, arithmetic, geometry and trigonometry. (2) The algebra will include the usual essentials through quadratics, with sets of linear equations, factoring, algebraic fractions with polynomial denominators, exponents other than positive integers and a few other topics receiving less attention than is now the practice. (3) The arithmetic will emphasize maintenance of previously acquired skills, renewed understanding, use of formulas, and better understanding of the use of approximate numbers in computation. (4) The geometry will be reduced to a few simple constructions, mensuration formulas informally established, a few propositions informally demonstrated, a brief treatment of similarity and use of the Pythagorean formula. (5) The trigonometry will remain numerical and will include simple applications of the sine, tangent and cosine ratios. (6) The names *albegra* and *general mathematics* will continue in use side by side with little and lessening distinction between books printed under the two titles. (7) There will be increasing stabilization of text and course content with occasional experimental digression. (8) Attention to better pupil understanding will increase with consequent lessening of emphasis on mechanical manipulation.

Is it to be concluded from this discussion and summary that I believe there is to be one course in ninth year mathematics taught to all classes alike? Such a conclusion is not intended nor would it account for all factors in the mathematical situation. A problem of great and increasing magnitude is that of necessary adjustment to interest, abilities and previous learning experiences of students. As previously indicated, the whole of the preceding discussion has been directed to the mathematics suitable for non-special students. For students of very low ability, little purpose or limited vocational needs (not that all these are by any means to be regarded as belonging to one class), there are a number of types of mathematics that can be given. But even among the non-special groups there is great variation in ability, purpose and interest that must be taken into consideration in the organization and teaching of mathematics. As indicated previously, I believe the adjustment will be less and less on the type of subject matter and will center on variations in emphasis. All courses will contain some algebra, some geometry, some arithmetic and some trigonometry. In some classes all of the traditional course in first year algebra will be taught (with modifications), a considerable amount of geometric constructions and informal establishment of mensuration (and possibly other) theorems, sufficient arithmetic for meaningful maintenance of skills and simple numerical applications of three trigonometric ratios. In other classes the work in algebra will be shortened, numerical trigonometry reduced to brief study of one function or omitted entirely, geometry reduced to informal establishment of mensuration formulas and the amount of arithmetic greatly increased to compensate for previous poor learning or rapid loss of skills. In some cases a third semester of mathematics should be given before students are permitted to take plane geometry to secure more mathematical maturity and make it possible for them to continue with sequential courses.

Thesis 12 of *The Second Report of the Commission on Post War Plans* deals briefly with the subject discussed in this article. I have here made no attempt to justify nor criticize that Thesis but to make a summary of certain facts that bear upon the problem there dealt with and to venture a forecast of trends for the immediate future.

# ◆ THE ART OF TEACHING ◆

## An Experiment in the Teaching of Geometry

*By* WYNETTE FOWLER
*Abraham Lincoln High School, San Jose, Calif.*

FOR THE school year of 1944–45, I was to have only geometry classes. I felt the need of some decided change in the methods of approach to the subject as well as in the results I had obtained before. The exasperation I had felt when students, who were perfectly capable of doing some original work, were satisfied to copy from the book or memorize what was given there led me to try a plan where the text was not given to each student, but used only as a classroom workbook. Thus, during the summer of 1944, after considerable research and study, I decided to combine some ideas from my study with those from my own experience into an outline for the course in geometry which I was to teach. In view of the results of the first year, the same plan was followed in 1945–46.

The basic desire as a result of this teaching outline procedure was that my students should have been helped to think clearly and logically while they learned the fundamentals of the subject matter of geometry.

A syllabus was mimeographed for the students which included originally 122 definitions, 14 axioms, 22 postulates, the listing of 33 constructions, and the statements of 115 theorems. The revision for the second year changed these figures to 132, 14, 27, 32, and 133 respectively. Of the 133 theorems, the proof of only 24 was required of the students, and these are tried over and over again until success is met. The remainder were either informally demonstrated or "discovered." The ideas of the theorems were learned and applied, but the greater weight of formal proof was eliminated in favor of practice and application.

A small library of geometry texts was gathered for classroom use including solid geometry, basic mathematics, and other texts. Lillian Lieber's *T. C. Mits*, the last two Yearbooks of the National Council of Teachers of Mathematics, and the author's own paper on "Approximate Computation" were available. A class set of two current texts was used (Seymour and Smith, *Plane Geometry* and Stone-Mallory, *Modern Plane Geometry*)

Perhaps someone will ask what was done for homework. A list of problems on a page could not be assigned. Thus, most of the homework was of a standing variety—studying and learning the definitions, axioms, postulates, and theorems discussed in class. Frequently, problems were outlined and left as homework, or simply a question asked at the end of the hour to be puzzled over for the next day. Actually, there was not too much homework given or done. Much of what is usually required proves to be cooperative. Then why not have cooperation under guidance? Even the review for a quiz or test was cooperative and done on class time. Even so, with September beginners, two full weeks were available in the first year for analytics, and five weeks in the second year for approximate computation and analytics.

The eight main topics planned and covered are: (1) Basic Concepts, (2) Triangles, Parallels, Rectilinear Figures, (3) Inequalities, (4) Circles, Loci, and Regular Polygons, (5) Proportion, Similar Polygons, and the Trigonometric Ratios, (6) Areas and Volumes, (7) Analytics, and (8) Approximate Computation.

A little of the actual class procedure should be indicated here so that the reader

may have some idea of it. The basic concepts are presented as words written on the board with class discussion of known meanings which are narrowed down to statements which are geometrically acceptable. The first of these is the word "solid." When that concept is understood, "surface" is considered and so on down to a "point." Where a word is apparently unknown to the group, it may be that breaking it up into parts, such as the word "con-centric," will develop the meaning.

After about 94 of these basic concepts are developed and understood, the uses of the compass and protractor are considered. Then we are ready for the basic constructions. Since some students have had mechanical drawing and most of them have had some elements of construction in junior high school mathematics, it seems to be a problem of clarifying student's suggestions rather than teaching new ideas here. We develop the three congruency postulates through constructions, and the axioms and other postulates develop as exercises with the basic concepts.

Thus, to this point, no proof has been required save the obvious or simple argument and demonstration—simply measurement and appearance. So, some of those deceptive exercises are required such as the first few problems of *T. C. Mits* which so entertainingly well point out the need for thought, plan, and order to arrive at valid conclusions.

Now, formally, the axioms are stated and demonstrated through algebra for they were first observed there. The other assumptions, postulates, are stated also at this time. As an exercise tying geometry to life in general we usually formulate the basic assumption of our form of government and note some of the consequences of this assumption.

Then, formal proof is approached through exercises which are cooperative board work with instructor prodding and questioning.

Briefly, this outlines the first unit of work. Perhaps if one is familiar with the Thirteenth Yearbook of the National Council of Teachers of Mathematics on *The Nature of Proof*, he will observe that the plan of my work is similar to the thesis there expressed, although I do not go quite so far as to let the student lead the way. Actually, I had developed my outline without reference to this work.

The Fifth Yearbook on *The Teaching of Geometry* gives a discussion of the Oakland, California, Plan. Perhaps any similarity between what I have done and this Plan is indirectly due to reading of the work of Miss Emma Hesse who was a member of that department.

Now, of course, when one sets out to try a new plan, he needs a means of checking the results. Thus, a standard test was chosen which aimed to be completely objective, required individual thinking as against pure memory work, and presented the type of situation in which the student is called upon to think through the proof of the main theorems of geometry. I found the Lane-Greene Unit Tests in Plane Geometry (University of Iowa, 1944 Edition) seemed to check these phases of achievement; and, being designed in six parts, provided a periodic check rather than a final check when it would be too late to do very much if failure was found to be the result of the teaching plan.

A very thorough analysis was made of the results of the tests. The median, true mean, and standard deviation were worked out for each new semester group and for the composite group. The median was used in reporting since it was the median and percentile scores the authors of the tests gave for comparative purposes. Several correlation coefficients were worked out. The algebra grade, age, class, and the Terman-McNemar I.Q. were studied for comparison of background. Except for the I.Q. this study did not indicate much more than that we at Lincoln have students from the first, or sophomore year through seniors in the same group with ages ranging from 14 to 19 years.

At the present time there are six groups

which may be considered and compared. A description of the groups follows with results of the tests presented.

Group A consisted of 23 students in another school (1942–43) where the text, Welchon and Krickenberger, *Plane Geometry*, was used in the usual manner. Homework assignments were regularly given and the older form of the same test was used. Their median I.Q. was 112.

Group B was a February beginning class of 31 students (1944) and taken over by the writer at the beginning of the second term. They had had several teachers in the first semester in two different groups, part using each of the texts listed above. They had homework assignments that first term. The first three tests were given after a review following the summer vacation. The tests were given at the proper point in the review which was carried on similarly to the plan of no text and no homework. Their median I.Q. was 112.

Group C, of 20 students (1944–45), also had another teacher for the first semester, used Stone-Mallory as the text, and had homework. The first three tests were administered by that teacher at the proper time. The second term these students were mixed in with those of Group D, although they were treated as an entirely separate group for comparison. Their median I.Q. was 115.

Group D, of 57 students (1944–45), might be considered the experimental group in that they were the first to follow the plan entirely for a year. Their median I.Q. was 112.

Group E, of 27 students (February beginners, 1945), followed the plan for the entire year, but required considerable review in the fall and was crowded to complete the six topics. Their median I.Q. was 111.

Group F, of 85 students (1945–46), has also followed the plan, but at a more rapid pace since there were five weeks left for analytics and approximate computation. They had a median I.Q. of 114.

The group indicated T in the following discussion consists of Groups B through F combined, 220 students. Group A was omitted as this summary was made to develop school norms. A point in desiring school norms is that in the June 1945 graduating class it was found that about 65% of the class went on to college of some kind. This would point out the fact that the school is somewhat selective in the first place and that norms such as given for the tests might not be good norms for this particular school. Perhaps this is borne out in the higher medians for 220 students.

Group T had a median I.Q. of 113. As can be observed, the median I.Q. varies only a few points from group to group.

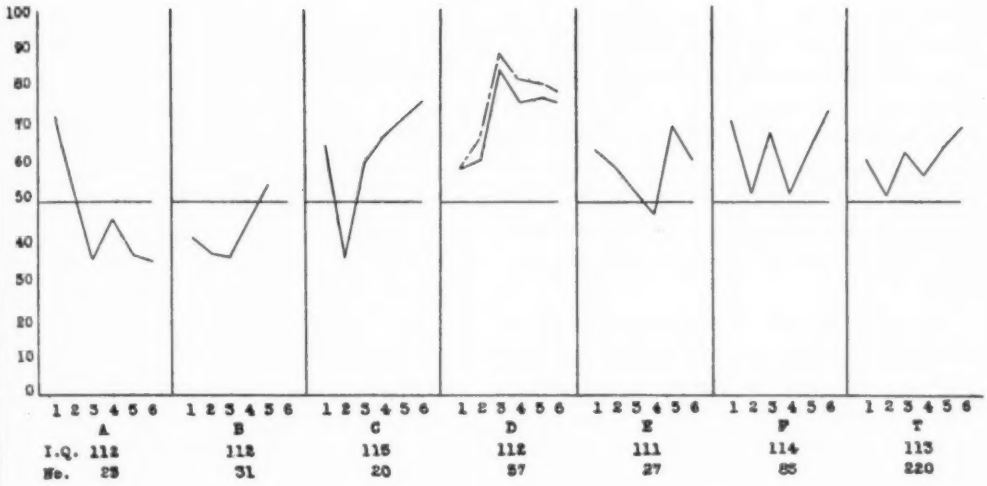
First, graphs (Graph 1) were made of the median percentile scores for each group and the total group. The graph for Group A clearly indicates why the writer was dissatisfied with a conventional approach to teaching geometry—the textbook-homework plan. In each of the graphs the horizontal line indicates the median percentile of the norms given for the tests.

Group B indicates that students with a decided handicap in the work of the first term were steadily improved through the plan.

Group C shows an interesting contrast between a textbook-homework procedure and that without either. This group was the most homogeneous as to age and class —all sophomores—and should have shown even better results, as they did in the second term under the plan.

Group D was by far the best group. Although they had a bit lower I.Q. median than Group C, their work was consistently better. This might lead to the conclusion that the plan was better than the textbook-homework procedure, but I feel that it was a case of industry on the part of the students and initial enthusiasm on the part of the teacher. The question arose at the time of the second test that perhaps the all-sophomore group was at a disadvantage since there were upperclassmen in Group D. The 28 sophomores of Group D were sorted out and the median score ob-

GRAPH 1




tained for each of the tests. This appears as the broken line on the graph for Group D—just a bit better than the whole of Group D. This subgroup's median I.Q. was 115—the same as all of Group C. It is interesting to note the leveling off to the same final score of Groups C and D.

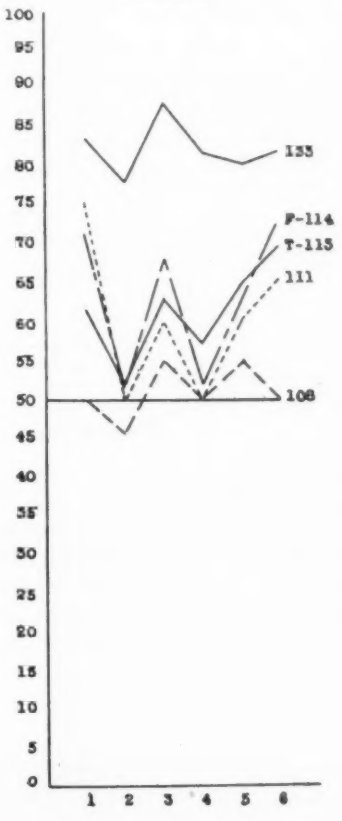
Group E shows lower scores than the experimental Group D, but they were again a February group which is a handicap. The tendency before and just after vacation was for lower scores. The sharp increase was only achieved through decided effort upon the part of the teacher.

Group F was taken over the work too rapidly—not intentionally, but perhaps developed in that there were fewer all-school interruptions such as bond drives.

In spite of Groups B and C with lower, or below standard, scores, it is evident from the graph of Group T that the whole school remains above the standard set by the authors of the tests through some 9,600 students' records.

As a means of showing what grouping according to I.Q., achievement in the first term of geometry, and size of class might do, Graph 2 was prepared. Without conscious intention most of the first term A and B students and higher I.Q.'s of Group F were regrouped into a single class of 23 students against 31 in each of the other two. These three groups are indicated by their I.Q. medians on the graph. The 133 I.Q. goes with the class of 23, and the 111 and 106 I.Q. go with the other two classes.

GRAPH 2



The composite score is that of group F. The total Group T is also indicated. Perhaps this quite definitely suggests some grouping of classes in the future. More will be said of this later.

The authors of the tests give coefficients of reliability ranging from .71 to .90 on the several tests obtained by the chance-half method and the Spearman-Brown formula and the Kuder-Richardson foot-rule formula.

There were some 55 students among the B through F groups who had taken the Revised Minnesota Paper Form Board Test. A correlation on the Pearson r-formula was worked out with this score against the sum of the raw scores on the first four tests, giving a coefficient of $r = 0.42$. This was better than that same test indicated for a correlation between their test and descriptive geometry scores. The correlation between I.Q. and the sum of the first four scores for these same students was $r = 0.48$.

For 134 cases including Groups A through E the correlation between algebra grades and geometry grades was $r = 0.68$. For the 220 cases the correlation between the sum of the first five tests and the I.Q. was $r = 0.33$.

Certain conclusions seem possible from this study. It is evident that better results were obtained in this school under the plan. I am satisfied to continue for the college preparatory geometry group in the same way, but the question arises as to whether some other form of geometry should not be offered for less capable students who do plan on college—the type of college which sets less high standards and is designed more for a continuation for two more years of general education. For that purpose, I am using the Iowa Plane Geometry Aptitude Test, previous mathematical achievement, the I.Q., and teacher recommendation for industry and application in the coming year and have chosen a text, Reichgott and Spiller's *Today's Geometry*, and developed a variation of the plan with no requirement of proof for that group which falls below standard.

At least, I see two outcomes of the two years of study. First, just the use of printed tests alone added motivation to the course—particularly when scores were posted and graphs made of progress along with remedial work immediately after the test. There is no argument that the teacher has been unfair on those test grades for the test and the norms are not of her making. Second, the teacher has been required to improve and augment her own efforts in setting objectives and evaluating the results in terms of the objectives.

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## Brotherhood Week, February 16-23, 1947

The National Conference of Christians and Jews announces the 14th annual observance of national Brotherhood Week to occur February 16–23, 1947. The theme is "Brotherhood-Pattern for Peace." Program aids for use in schools and colleges may be secured by writing to the National Conference of Christians and Jews, 381 Fourth Avenue, New York 16, New York. Materials are adapted to age levels in the schools. Plays, comics, posters, book lists and other types of literature and visual aids are available.

# EDITORIAL

## Elimination of Obsolete Material in Mathematics

SINCE the advent of World War II mathematics has regained a great deal of respectability among groups who formerly were convinced that it was of little, if any value, as it has been traditionally presented. Those men and women who used it in their military careers will likely tell their younger brothers and sisters to study their mathematics in the schools so as to be better equipped than the previous generation. However, there will be some people, in fact there are already some people, who will seriously question the value of mathematics beyond the mere fundamentals of arithmetic for most people. Now, while mathematics seems to be of some importance to the well educated citizen, is the time for teachers and textbook writers to omit as much of the "deadwood" from the course as is possible. This is particularly true in the more elementary parts. Many courses in algebra as such could be greatly improved to say nothing of the need for modernizing the organization of formal geometry if only some of our authors would really think about how serious the need for revision is.

Take the topic of factoring, for example. No one who makes a careful study of the situation can any longer justify the tea hing of any other types of factoring in the ninth grade beyond taking out a common monomial factor, the factors of a perfect square, and possibly the difference of two squares. No one can find a realistic application of factoring a trinomial of the form $ax^2+bx+c$. Someone may say that such skill is needed to solve a certain quadratic equation, but the truth is that no quadratic equation that results from an attempt to solve a practical problem can be solved by factoring. It must be solved by the method of completing the square or by the quadratic formula. In geometry there is only one possible use for a knowledge of a new factoring type, and that is late in the course when and if Hero's formula is deduced. Moreover, many more or less complicated operations on polynomials and equations are no longer of any real importance and should be relegated to the algebra museum. The main use of algebra is the formula. Algebra should be taught as a way of thinking, and unless pupils are taught to understand what they are given in algebra and geometry we may expect the whole structure to crumble insofar as many pupils are concerned. One of the commonest complaints of pupils who have graduated from the high school and who did not understand the subject even though they received a passing mark is that they find no way to use it in their lives. Just so! How can one make any intelligent use of knowledge which he does not possess. We must put less emphasis on the mere mechanical or manipulative aspects of algebra and give more attention to meanings as we are asked to do in arithmetic. It is high time that we all give some real thought to better kinds of content and ways of teaching it to pupils, throughout the elementary and secondary schools.—W. D. R.

# ◆ IN OTHER PERIODICALS ◆

*By* NATHAN LAZAR

*Midwood High School, Brooklyn 10, New York*

*The American Mathematical Monthly*

November 1946, Vol. 53, no. 9

1. Bergman, Stefan, "Models in the Theory of Several Complex Variables," pp. 495–501.
2. Beckenbach, E. F., "An Inequality of Jensen," pp. 501–505.
3. Beaman, Elizabeth, "The Moduli of the Roots of an Algebraic Equation," pp. 506–510.
4. MacQueen, M. L., and Hartley, R. W., "Elliptic Euleroids," pp. 511–516.
5. Halteman, A. E., "A Geometric Approach to the Covariants of a Cubic," pp. 517–520.
6. Discussions and Notes, pp. 521–525.
   (a) Brauer, Alfred, "On the Exact Number of Primes below a Given Limit."
   (b) Goormagtigh, R., "On the Orthopole."
   (c) Molina, E. C., "Fermat's Theorem as a Problem in Probability."
7. Clubs and Allied Activities, pp. 526–528.
8. Recent Publications, pp. 529–531.
9. Problems and Solutions, pp. 532–541.
10. News and Notes, pp. 541–542.
11. General Information, pp. 543–547.
    (a) The Watson Scientific Computing Laboratory.
    (b) New Preliminary Actuarial Examinations
    (c) Revised Selective Service Memorandum no. 115.

*School Science and Mathematics*

December 1946, Vol. 46, No. 9

1. Muuss, Elsa, H., "Of Millions and Millionths," pp. 795–799.
2. Kearney, Dora E., "Improving Language Ability in Mathematics," pp. 800–802.
3. Peak, Philip, "The Slide Rule in Junior High School Mathematics, pp. 821–824.
4. Bain, Sherwood C., "The Mathematical Problem of High School Physics," pp. 846–854.
5. Niessen, A. M., "Illustrations of the Versatility of the Binomial Theorem," pp. 855–860.
6. Posey, L. R., "Change of Base," pp. 871–878.
7. Nyberg, Joseph A., "Notes from a Mathematics Classroom," pp. 879–882.
8. Book Reviews, pp. 888–890.

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1. Bedford, F. L., "Designing the Mathematics Classroom," *American School and University*, 1946, pp. 199–202.
2. Crosmo, C. D., "Hidden Half of the Problem," *Journal of Engineering Education*, 27: 149–152, October, 1946.
3. Deans, E., "Meaningful Number Work," *Instructor*, 56: 24+, November, 1946.
4. Fleischman, L., "Mechanical Aids for the Solution of Linear Equations," *American Journal of Physics*, 14: 326–327, September 1946.
5. Grossnickle, F. E., "Illustrations in Arithmetic Textbooks," *Elementary School Journal*, 47: 84–92, October 1946.
6. Guiler, W. S., "Difficulties Encountered in Percentage by College Freshmen," *Journal of Educational Research*, 40: 81–95, October 1946.
7. ———, "Hard Addition Facts: a Chart," *Instructor*, 56: 25, November 1946.
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10. Johnson, E. A., "Arithmetic Test for Grammar Grades," *Grade Teacher*, 64: 6+, November 1946.
11. Manion, C. E., "Mathematics of History," *Teachers College Journal*, 18: 2–3+, October 1946.
12. Rosenberg, R. R., "What Shall We Teach in Business Arithmetic," *Business Education World*, 27: 74–76, October 1946.
13. Shores, J. H., "Why Children Dislike Arithmetic?" *Educational Administration and Supervision*, 32: 357–363, September 1946.
14. Sturges, H. A., "Arithmetic Methods," *Sierra Education News*, 42: 9, November 1946.

# An Inquiry—What's Going on in Your School

Please copy, fill out this page and mail today.

The Commission on Post-War Plans of the National Council of Teachers of Mathematics would like to get a picture of what is happening in mathematics in typical schools—large and small. You can help by providing the information requested for your building or better still, for your whole organization if you are part of a city system. It is assumed that a report of trends will, in due time, be published in THE MATHEMATICS TEACHER. In any case a mimeographed report will be mailed to all who contribute to this inquiry. Feel free to be altogether frank—the report will in no way refer to you or your school without getting your special permission. Parts II and III of this Inquiry will be published in the April and May numbers of THE MATHEMATICS TEACHER. Watch for them.

Part I

Name of School _____
Location of School _____
Name of person reporting _____
Address _____

1. Compare the attitude of administrators toward mathematics with that before the war. Is it now (a) more favorable, (b) less favorable, or (c) about the same.
2. What grades are enrolled in your building? _____
3. Please fill in the following table:

| Grades | Pupils in Grade | Pupils Enrolled in a Mathematics Class |
|---|---|---|
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| 11 | | |
| 12 | | |

4. As concerns your seventh and eighth grade courses in mathematics:
   a. Would you describe these courses as general mathematics? _____
   b. What per cent is devoted to informal geometry? _____
   c. What fraction of pupils entering the seventh grade are adequately prepared in their mastery of common and decimal fractions? _____
5. Do you provide a double track in the ninth grade? _____
   If so what per cent of your ninth grade pupils take:
   a. algebra? _____
   b. General mathematics? _____
   c. commercial arithmetic? _____
   d. shop or industrial mathematics _____
   e. other courses (specify)? _____
6. Do you have "ability grouping" in the ninth grade? _____
7. Do you provide a double track in the 10th grade? _____
   If so, what per cent of your tenth grade pupils take:
   a. first year algebra? _____
   b. first year general mathematics? _____
   c. second year general mathematics? _____
   d. demonstrative geometry? _____
   e. commercial arithmetic? _____
   f. other courses (specify)? _____
8. Would you welcome the publication of a pamphlet providing guidance to your pupils as to the mathematics needed in various vocations and professions? _____
9. Part II of this inquiry will appear in the April number of THE MATHEMATICS TEACHER. If you have any questions that you would like to see in the inquiry be sure to send them with this report.
10. Feel free to supplement your responses to the preceding inquiry by further comments on separate pages. As stated earlier, the main idea is to picture the present mathematics picture in your school or your building.

When you have filled out the blanks, which we hope will not take more than 10–15 minutes, mail to Raleigh Schorling, Chairman, Commission on Post-War Plans, University High School, Ann Arbor, Michigan.

# BOOK REVIEW

*An Introduction to College Mathematics.* By C. V. Newsom, Prentice-Hall, Inc. New York. 7+344 pages. $3.75.

The problem of presenting a suitable freshman course in mathematics for the college student who plans to take only one course in college mathematics has been considered by many college teachers in the past 15 or 20 years. One of the results has been the publication of textbooks designed for such a course, the most recent of these being C. V. Newsom's *An Introduction to College Mathematics.*

Books of this so-called "cultural type"[1] attempt to present more of the ideas of mathematics and their applications and cultural implications and are not bound by the traditional content of courses in algebra, trigonometry, analytic geometry, etc. Hence they vary in the topics covered as well as in the method of treatment of each topic. Professor Newsom states in his preface that "No topic has been chosen simply because of some special appeal to the author or because of a traditional prejudice among mathematicians in favor of it. In fact, all material introduced was examined critically for its possible value to the non-specializing student. . . . Considerable material proposed for the course was discarded when it was found that students generally were unable to appreciate its significance." These criteria may account for the omission of certain interesting and valuable topics which have been proposed for this course and have been included in other "cultural type" texts. These omissions cause this book to lack the breadth found in some of the other similar books. The topics included are well indicated by the chapter headings: 1. The Nature of Mathematics, 2. Number and the Operations of Arithmetic, 3. The Arithmetic of Numbers in the Exponential Form, 4. The Arithmetic of Measurement, 5. Logarithms, 6. Some Topics in the Mathematics of Finance, 7. Progressions of Numbers, 8. Combinations and Probability, 9. Functional Relationships, 10, Variation, 11. The Circular Functions, 12. The Equation, 13. Some Common Curves.

Important ideas are often developed in the illustrations and exercises rather than through detailed explanation. Certain exercises are marked with a dagger to indicate that they must not be omitted since they are important to the development of the text. The exercises are planned with care and indicate a successful effort to find new practical problems. Careful assignment of the exercises and subsequent discussion of them will enrich the classwork considerably.

The sparing use of bold face and italic type, and the relatively few formulas developed make the page look less formidable than in many other texts. The informal presentation and the liberal use of historical background to amplify many topics helps to keep a student's interest in the subject. However, it is difficult to develop all parts of a book free from conventional presentations. On the whole, Professor Newsom attacks new topics in a fresh manner but some chapters, such as Chapter 5, lapse into the conventional.

New topics are often introduced in an abstract and concise manner and later the topic is made more concrete and practical. In many places the explanation of a topic and its applications is given very briefly but, as is anticipated in the preface, the instructor may desire to supplement the written material.

Professor Newsom states that "the order of topics is the result of much thought and experimentation and is regarded by the author as pedagogically sound." A new order of topics always seems a little strange and cannot be evaluated satisfactorily without actual trial. For example, it is somewhat surprising to find annuities discussed before geometric progressions. This forces the studnt to solve each annuity problem by a method similar to the usual method of deriving the formula for the sum of a geometric progression and then carry out the computation by logarithms. Although it is remarked that this method may seem long and involved, there is no mention of the shorter method using tables of $s_n$ and $a_n$. Whether this presentation will make the basic ideas clearer or whether the student will get bogged down in the manipulations can be determined only by an actual trial. Similarly one must try the method of graphical solution of equations by means of intersection of two curves in Chapter 12 to see whether it is preferable to the usual method of finding the intersections with the $x$-axis.

Those college teachers who agree with Professor Newsom that "no single one of the traditional courses is suitable" for those college students who do not expect to do extensive work in mathematics will welcome this book. The content of a freshman course for such students must necessarily be determined by trial. Each careful trial, such as this book, "which has been developed during the past twelve years," helps to determine the mathematics which should be included in a course for the largest group of college students—those who do not expect to do extensive work in mathematics.—HOWARD E. WAHLERT.

[1] Brown, K. E., "What is General Mathematics?" THE MATHEMATICS TEACHER, vol. 39, #7 (Nov. 1946), p. 329.

# NEWS NOTES

American Christmas customs stem largely from Pagan rites developed long before the birth of Christ, according to a new article prepared for the *Encyclopaedia Britannica* by Mary Emogene Hazeltine, principal of the University of Wisconsin Library School.

Of American celebrations of Christmas, the custom of exchanging gifts is one of the few that springs from the Christian era, Miss Hazeltine writes. Christmas candles, bonfires and Yule logs arise from the heathen festivals observing the winter solstice, December 21, when the days begin to lengthen. That was a time of great rejoicing for ancient sun-worshippers who built bonfires to give strength to the "winter sun-god."

Pre-Christian Romans ornamented their homes with green boughs and flowers for the Feast of the Saturnalia which began on December 19. Druids gathered mistletoe for that season, while ancient Saxons used holly, ivy and bay. Modern mistletoe and holly wreaths come from those peoples.

Christmas trees, too, spring from ancient German days when the Germanic tribes made sacrifices to the sacred oak tree of Odin, the *Britannica* article states. When the missionary St. Boniface traveled to Germany in the eighth century and saw these celebrations he persuaded the natives to substitute a fir tree for the oak and to adorn it in tribute to the Christ child. Thus was born the first true Christmas tree. Prince Albert introduced the tree to England after his marriage to Queen Victoria, while it was brought to America with all its decorations and spangles by German immigrants to this country.

In the pre-Christian world the celebration of the winter solstice was the principal event of the year. And because the exact day and year of Christ's birth have never been satisfactorily settled, the fathers of the Church, when they decided in 340 A.D. upon an official date for the birth of Christ "wisely chose the day of the winter solstice which was firmly fixed in the minds of the people and which was their most important festival. Owing to changes in man-made calendars," Miss Hazeltine continues, "the time of the solstice and the date of Christmas vary by a few days."

The exchange of Christmas gifts, according to the *Britannica* article, was originated by a bishop, born in the third century A.D., who was later made the Netherlands' patron saint of children—San Nicolaas. Santa Claus, as the American contraction of his name calls him, had a reputation for making secret gifts during the Christmas season. Saint Nicholas has become a familiar figure in the folklore of many lands, yet in the Netherlands, as in Belgium, gifts are given on December 6, a special holiday for children, instead of on Christmas day. In France, too, gifts are exchanged on New Year's instead of Noel, and in Italy presents are distributed on the Epiphany, the celebration day of Christ's baptism.

St. Francis of Assisi is responsible for the birth of two widespread Christmas customs, Miss Hazaltine points out in the new article. He originated the use of the creche, a miniature tableau of the Nativity, in 1224, and since then the manager scene is the heart of the Roman Catholic celebration. St. Francis also encouraged the singing of Christmas carols, and they passed from Italy to France and Germany and then to England and the United States, always retaining their fervent and festive religious style.

During the Middle Ages the entire Christmas season was celebrated in equal fervor with that now shown on Christmas day. In some lands the celebration lasted from December 24 to January 6—Twelfth Night—and in others from December 21 to February 2. By the year 1644 Christmas in England had become such a wild orgy that the Puritans forbade its celebration by law.—The *Encyclopaedia Britannica*.

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Miss Elizabeth McHenry, Arkansas State Representative for the National Council, has resigned her teaching position at Little Rock, Arkansas, and has accepted a teaching position with the University of Tennessee.

Mr. J. H. Banks, State Representative in Alabama, has resigned his teaching position at East Central Junior College, Decatur, Mississippi, and has accepted a teaching position in the State Teachers College in Florence, Alabama.

Miss Dora E. Kearney has resigned her position as Assistant Professor in the Mathematics Department in Iowa State Teachers College, Cedar Falls, Iowa. Miss Kearney is Vice-president of Iowa Association of Mathematics Teachers and Iowa State Representative for the National Council of Teachers of Mathematics. She received her masters degree at the University of Minnesota where she was a member of Phi Beta Kappa.

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The Women's Mathematics Club of Chicago and Vicinity held its last meeting on November 2, 1946 at Mandel's Tea Room. The speaker was Professor Walter H. Carnahan whose topic was "Mathematics Is Where You Find It."

Professor Carnahan is counselor in mathematics in the State Department of Public Instruction for Indiana and on the staff of Purdue University. A part of his work is to build up the interest of pupils in high-school mathematics and to work for better organization and teaching of the subject. He broadcasts on mathematics on the average of two Mondays out of every three throughout the school year over station WBAA. He has lectured to about 75,000 high-school pupils in Indiana in about 400 high schools ex-

clusively on mathematics. He is past president of the Central Association of Science and Mathematics Teachers, on the Board of Directors of the same organization as well as on the Board of The National Council of Teachers of Mathematics. He has taught in the public schools for thirty-three years.

Teachers are welcome to attend all meetings of the club, regardless of membership. If they wish to join, the dues of $1.00 a year may be paid at the meeting. National Council dues of $2.00 may also be paid through the treasurer.

*President*, Mrs. Florence B. Young, Steinmetz High School
*Vice-President*, Miss Edith Levin, Englewood High School
*Secretary*, Mrs. Janette S. O'Connor, Chicago Heights
*Treasurer*, Miss Edna M. Feltges, Wilson Junior College
*Program Chairman*, Miss Martha Hildebrandt, Proviso Twp. High School

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The Association of Mathematics Teachers of New Jersey held its 83rd annual meeting at the Hotel Dennis in Atlantic City on Saturday, Nov. 9, 1946.

PROGRAM

*Theme:* TOOLS AND TECHNIQUES IN THE TEACHING OF MATHEMATICS

10:30 A.M. Presidential Greetings
Madeline D. Messner, Abraham Clark High School, Roselle, New Jersey.

10:40 A.M. Experiences in Teaching Fractions
Mrs. Euphemia Gibson, Roosevelt Elementary School, Kearney, New Jersey.

11:10 A.M. The Consumer Figures
Hubert B. Risinger, Davey Junior High School, East Orange, New Jersey.

11:40 A.M. Experimental Developments in Twelfth Year Mathematics
Ralph C. Miller, Ridgewood High School, Ridgewood, New Jersey.

12:10 P.M. Business Meeting and Discussion.

1:00 P.M. Luncheon.

2:00 P.M. Professor John R. Kline, Head of the Department of Mathematics, University of Pennsylvania, Philadelphia, Pennsylvania.

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The Eastern Division Mathematics Section of the Colorado Education Association held its biannual meeting October 25th at Denver, Colorado.

At 9:30 A.M. a short business meeting preceded an interesting talk by Dean O. H. Rechard of the University of Wyoming. The title of his address was, "Leaves From a Mathematics Teacher's Scrapbook." The meeting was then given over to a discussion of College Entrance Requirements. The Speakers included Professor Aubrey J. Kempner of the University of Colorado, Mr. Russell Casement of South High School in Denver, Mrs. Lester E. Griswold who spoke as a parent, and Doctor Elmer C. Darling of the Veterans Administration. The rest of the meeting was in the form of a general discussion.

A Fellowship Luncheon closed this very interesting and stimulating session. Approximately one hundred fifty persons were in attendance.

The officers are:
Kenneth E. Gorsline, Pres.
Aubrey J. Kempner, Vice-Pres.
Burnett Severson, Sec.-Treas.

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Word has just been received of two additional institutes held in the Los Angeles Region in October and November. Further information can be secured by writing Mr. E. A. Sundberg at Polytechnic High School, 400 West Washington Blvd., Los Angeles.

"Planning a Mathematics Curriculum to Meet Needs of ALL Secondary Students"
Cornelius Siemens, Polytechnic High School, Los Angeles.

"Improvement of Arithmetic Instruction in the Junior High School"
Peter L. Spencer, Polytechnic High School, Los Angeles.

Formerly announced dates were:

Nov. 9–9:30 A.M.—Los Angeles County Schools Office, 808 N. Spring Street, Los Angeles, "Using Audio-Visual Tools in Mathematics"
Mrs. Thelma Clark and Mrs. Emma Sturtevant.

Dec. 7–9:30 A.M.—Lincoln High School, 3501 North Broadway, Los Angeles, "Significance for Mathematics Teaching of War Experiences and Recent Historical Research," Dr. Morgan Ward.
Section meetings on visual aids.

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PROGRAM OF THE SAN DIEGO MEETING

October 5, 1946

Place: Campus Elementary School of San Diego State College

9:30 A.M.—*Gaining Basic Mathematical Understandings*
Presiding: Mr. Bruce McLean, Director of San Diego Region of California Mathematics Council
Speaker: Dr. Richard Madden, Director of Teacher Training, San Diego State College
Discussion Sections:
Grades 1–8: Mr. Dan Dawson, Principal, Cabrillo School, San Diego
Grades 9–12: Mr. Richard Wooten, Secondary Coordinator, San Diego County Schools

11:30 A.M.—Business meeting of state council (Nomination of 1947 officers)
Presiding: Miss Harriette Burr, President

12:00 M.—Luncheon
Presiding: Miss Rachel Keniston, Program Vice-President of Council
Topic: *Current Problems in the Teaching of Mathematics*
Speakers: Officers of the California Mathematics Council

1:30 P.M.—Symposium on *Understanding Mathematics through Audio-Visual Aids*
Presiding: Mr. Bruce McLean

Speakers:
Dr. Grace Fernald, U.C.L.A.—"Audio-Visual Aids to Remedial Techniques in Arithmetic"
Mr. Dale Carpenter, Mathematics Supervisor, Los Angeles City Schools—"Visual Aids Used in the Los Angeles Schools in Mathematics"
Dr. Cornelius H. Siemens, Director, Compton Junior College "Improved Audio-Visual Aids in Mathematics—How soon Will They Be Here?"
Discussion leader: Dr. Richard Madden

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The Mathematical Section of the Maryland State Teachers Association, held a luncheon meeting on October 18, 1946 at the Emerson Hotel, with the Chairman Mrs. Edna T. Price, presiding.

Following the luncheon, attended by 121, an additional 50 or 60 more came in to hear the numbers rendered by 100 girl members of the Southern High School Glee Club and attend the business meeting at which time the following officers were re-elected:
*Chairman*, Mrs. Edna T. Price, Baltimore, Md.
*Co-Chairman*, Miss Margaret Bowers, Elkton, Md.
*Treasurer*, Mr. Arnold Ortmann, Baltimore, Md.
*Asst. Treasurer*, Miss Ruth Warren, Randallstown, Md.

Miss Margaret Hamilton of Cumberland was elected to succeed Miss Paterson of Baltimore as secretary.

"Greetings" were extended the group by Dr. Kelso Morrill, of Johns Hopkins University.

The speaker of the day was Dr. Virgil Mallory, State Teachers College, Montclair, New Jersey and his talk was on the work of the Commission on Post War Plans in mathematics. Dr. Mallory did not give a formal title to his talk.

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Dr. Richard E. Gadske is back in the Mathematics department at the New Trier Township High School in Winnetka, Illinois after serving 4 years in the Navy. He served as a Commander in the Bureau of Naval Personnel, Washington, D. C. during his last year of service. He received several commendations.

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A general meeting of The Mathematical Association with President S. Chapman presiding was held at the Polytechnic in London on April 24 and 25, 1946. The program follows:

WEDNESDAY, APRIL 24

10:30 A.M. Business meeting
1. Annual Report of the Council for 1945.
2. Election of Officers and the Council.
3. The revised regulations for the Teaching Committee and the new Committee were announced.

11:00 A.M. Presidential Address by PROFESSOR S. CHAPMAN, D.Sc., F.R.S.
"The University Training of Mathematicians."

2:00–4:00 P.M. Discussion
"Geometrical Drawing in the H.S.C. Course in Schools."
To be opened by DR. W. G. BICKLEY, followed by DR. S. WEIKERSHEIMER.

Opportunities of combining action with thought are rare in the Mathematics Course, and should be seized, or, indeed, created, wherever possible. Geometrical drawing provides such opportunities and emphasis on it should be maintained, especially in the post-school Certificate stage. The subject is educative in the best sense (e.g., descriptive Geometry and geometrical perspective), and can be made to interest most boys and girls. Last—and *not* greatest—it has considerable practical value.

The Teaching of Descriptive Geometry (incl. Geom. Drawing).
1. The oblique projection of a square, the oblique and orthogonal projection of a prism, this projection as a silhouette of parallel rays of light.
2. Theorems on the position of straight lines and planes in space.
3. Problems on point, straight line, and plane.
4. Intersection of plane-surfaced solids with straight lines and planes.

4:45 P.M. Relaxation Methods in Engineering.
MR. D. N. DE G. ALLEN.

The relaxation method was originally devised in order to determine the stresses in a loaded framework, and it enables large numbers of simultaneous equations to be solved speedily and to any required degree of accuracy. The method was later developed as a means of obtaining particular numerical solutions of partial differential equations, and has been successfully applied to a wide range of problems in the War years. These include applications in Hydrodynamics, Elasticity and Plasticity, and no insuperable difficulty has been offered in the solution of various non-linear partial differential equations or of simultaneous equations. Problems involving eigen-values have also provided an interesting and profitable source of problems for investigation.

THURSDAY, APRIL 25

10:00 A.M. Statistical Science and Agricultural Research.
MR. D. J. FINNEY.

An account of the part played by modern methods of statistical analysis in Agricultural research, with particular reference to the work of the Statistical Department at Rothamsted Experimental Station. The paper will be chiefly concerned with the development of the field plot trial from the original "simple" plan of assigning one plot to each of a miscellany of treatments to the complex factorial and replicated experiments used to-day. Various types of experimental design will be described and the appropriate methods of statistical analysis discussed.

11:00 A.M. A new Approach, by Means of Isogonal Conjugates, to Certain Geometrical Theorems and to a General Theory of Conics.
MR. H. E. PIGGOTT and DR. ALFRED STEINER.

Use of the circumcircle in connection with the triangle and certain notable points. Isogonal conjugates. The symmedian point and the Brocard points. The cosine, Lemoine and Brocard circles (Mr. Piggott).

Isogonal conjugates in trilinear co-ordinates. Conics circumscribing a triangle constructed as the locus of isogonal conjugates of a straight line. Special case of the rectangular hyperbola through orthocentric points. Steps towards a general theory. Generalisation of the properties of notable points and circles connected with the triangle of reference.

2:00 P.M. The Place of Mathematics in Secondary (Modern) Schools.

A discussion to be opened by MR. F. J. SWAN, followed by MR. C. T. LEAR CATON and MR. H. WATTS.

The Secondary Modern School—organization—choice of pupils.

Problems arising from (a) the raising of the school age; (b) the possibility of multilateral schools or a second break at 13+.

General questions of control, staff, equipment and curriculum.

Development of a purposeful educational philosophy. The place of mathematics in an integrated scheme.

The principles which should guide the teacher in the choice of material and manner of presentation. The need for co-ordinated experimental work in connection with the curriculum.

4:45 P.M. The History and Aims of the *Mathematical Gazette*.

PROFESSOR T. A. A. BROADBENT.

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Arthur J. Crowley, Director of the Education Department of Popular Science Publishing Company and *Reader's Digest* announces the appointment of Dr. David J. Goodman, an educator of outstanding accomplishments and long experience, as Editor-in-Chief of the new Audio Visual Department of Popular Science Publishing Company, New York, N.Y. This division has been created to promote the development and sale to schools and colleges of audio-visual education aids on a nationwide basis. The department's program will be one of gradual expansion and in time it will handle all of the major audio-visual aids to learning, involving all levels of education.

Dr. Goodman's solid background and splendid accomplishments thoroughly qualify him for his new post. While at New York University, he majored in Administration and Supervision of Audio-Visual Education. His Doctor's Thesis is entitled, "Comparative Effectiveness of Pictorial Teaching Materials" (Motion Pictures vs. Strip Films). He has had nine years of full-time experience in public school work in the preparation of audio-visual curricula and the direction of audio-visual instruction. He has also had extensive experience in editorial work, still and motion picture photography and recording.

Dr. Goodman has had rich and varied experience in his chosen field. His background includes position as Editor for Young America Films, Inc., N.Y.C.; Department Editor for Educational Screen, Chicago, since 1941; Educational Supervisor for New York City Board of Education Program on the Development of Objective Teaching Materials and Techniques. During the war for a two-year period he was Educational Specialist for the Army Air Forces, Training Aids Division, in charge of preparation of Instructors' Guides to accompany Army Air Forces training films and filmstrips. He conducted a very extensive survey in the utilization of filmstrips.

He has contributed numerous articles to educational journals such as: "Journal of Educational Research," "Safety Education," "Journal of Education," "Film and Radio Discussion Guide" and others.

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There will be a reunion breakfast for all those who have attended the Mathematics Institute at Duke University, at the Atlantic City meeting of the National Council on Saturday, March 1st. Make your reservation with Veryl Schult, Wilson Teachers College, Washington 9, D. C. Details will be announced at the meeting Friday night, Feb. 28.

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The annual meeting of the Association for Supervision and Curriculum Development will be held March 23 through 26, 1947 at the Sherman Hotel, Chicago, Illinois.

All persons interested in improving instructional programs will attend. Special emphasis will be given to making this meeting an actual laboratory for learning in various group processes. There will be a large number of small discussion groups dealing with issues in education as implications for curriculum change in this modern technological world; human relationships in the supervisory processes; and building curriculum based on child needs and development. The general evening sessions will give attention to problems of current concern on curriculum planning. All educators interested in providing better school programs are invited to attend.

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The Nassau County Mathematics Teachers Association held a meeting on November 21 at Southside High School, Rockville Centre, New York.

George C. Campbell, Supervisor of the Actuarial Division of the Metropolitan Life Insurance Company, was the speaker.

He developed some of the theory of compound interest, using geometrical progressions to obtain the formulas for present values of annuities. He also discussed the derivation of mortality tables and recent changes in them.

The President, Miss Alice Griswold of Garden City High School presided at the meeting and Mr. John Clark of Sewanaka High School, Floral Park, Vice-President and Chairman of the program committee, introduced the speaker.